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Combinatory System Theory

A theory for understanding and controlling collective phenomena

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Abstract

This study aims to present a simple theory - the Theory of Combinatory Systems - which is able to describe, interpret and explain many collective phenomena and their observable effects.
We define as combinatory *those systems whose agents are relatively similar agents, each of which produces a micro behaviour similar to that of the others. The macro behaviour of the system, as a unit, derives from the combination of the analogous behaviours of its similar agents, according to a* feedback relation.
A necessary and sufficient condition for a collectivity (observable or hypothesized) to be considered a combinatory system is the existence of a feedback *between the* micro behaviour *of the individuals and the* macro behaviour *of the collectivity constituting the system*
The Theory of Combinatory Systems searches for the conditions that produce the macro behaviours and proposes models to interpret the collective phenomenon.In particular, the Theory focuses on the necessity to understand the nature of the macro rules, which specify the recombining *factor, and of the macro rules, which specify the* necessitating *factor; the joint action of these factors gives rise to and maintains the macro and micro behaviours.*

If we classify combinatory systems according to their macro behaviour (or their macro effect) we can, despite the variety of phenomena produced, determine five fundamental types of combinatory systems, the most important of which are:

1. *systems of* accumulation, *whose activity involves an* accumulation of objects*, behaviours, states, or effects of some kind;*
2. *systems of* diffusion, *which have as their macro effect the diffusion of an «object» (of a feature, a particularity, or a «state») from a limited to a greated number of agents of the system;*
3. *systems* of pursuit, *which produce a behaviour consisting in a gradual shift of the system toward an objective, a limit, a target, just as if the system, as a single entity, were pursuing a goal or trying to move towards ever more «advanced» states;*
4. *systems of* order, *which produce a phenomenon that can be interpreted as the attainment or maintainment of an arrangement, an ordered disposition, or a form of some kind, among the agents that form the system*
5. *systems of* improvement *and* progress: *their effect is to produce progress (according to commonly accepted value judgments) in the overall state of a collectivity in which the individuals pursue their search for individual improvement (that is, an increase in some parameter judged to be useful or favorable).*

The second aim of this study is to illustrate the systems of improvement *and* progress, *which are the most relevant class of combinatory systems, and to present three typical classes: - Medial systems - Maximal systems - Minimal systems of improvement and progress.*

Key words: Combinatory Systems, Micromotives and macro behaviour, Dynamic Systems, Populations and collectivities, Systems of *improvement* and *progress,* Systems of *accumulation,* Systems of *diffusion,* Systems of *pursuit,* Systems of *order*

1 - The Combinatory Systems Theory

1.1 Introduction. The simplest definition

The first aim of this paper is to present a simple theory – the *theory of combinatory systems* – which (I hope) is able to describe, interpret and explain many *collective* phenomena and their observable effects. The second aim is to illustrate the most relevant class (I think) of combinatory systems: the systems of *improvement* and *progress* [1].

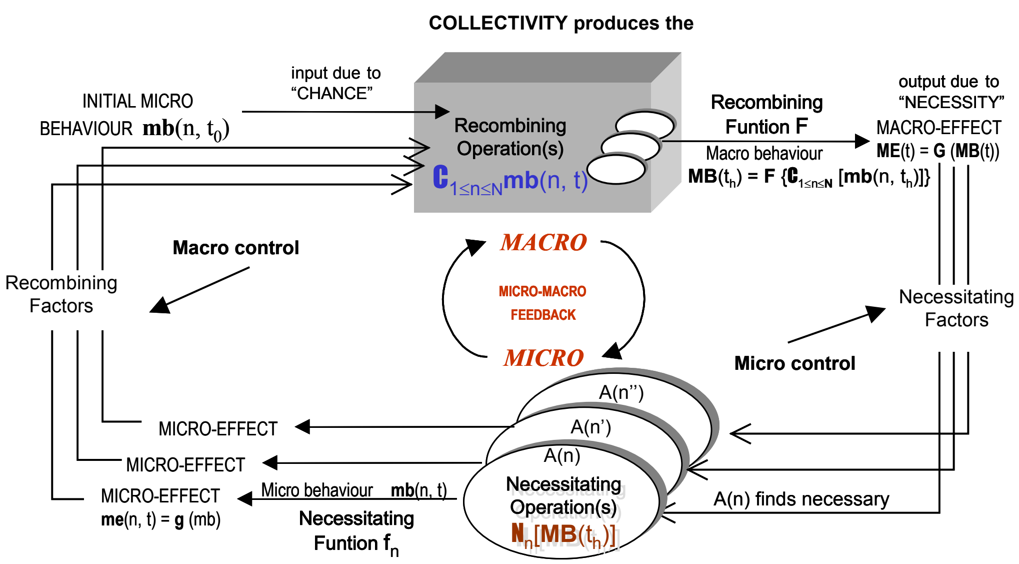
In plain words, I define as (social) *combinatory systems* the *unorganized systems* made up of a collectivity of *similar agents*, each of which produces a *micro behaviour* and a *micro effect* similar to that of the others. The *macro behaviour* – and/or the *macro effect* – of the system, as a unit, derives from the *combination* of the analogous behaviours – or effects – of its similar agents, according to a *feedback* relation between micro and macro behaviours [2].

This internal feedback between micro and macro behaviours – or between their micro and macro effects – guarantees the maintenance over time of the system's dynamics and produces some self-organization effect. When the system starts up "by chance" it then maintains its behaviour "by necessity", as if an Invisible Hand or a Supreme Authority regulated its time path and produced the observable effects and patterns (figure 1).

There is nothing strange here: the *invisible hand* is nothing other than the micro-macro feedback action.

The feedback arises from *necessitating factors (*which act on the agents in the system) and is maintained by the action of *recombining factors* (which act on the collectivity).

Fig. 1 – The micro macro feedback



1.2 The formal definition

In order to give a simple illustration we shall indicate by **S**(t, **N**) = [t, A(1, t), ..., A(n, t), ... , A(**N**, t)] a non-ordered system formed by **N** agents (or elements), A(n, t), $1 \leq n \leq \mathbf{N}$, observed for $t \in T$, appropriately defined [3].

Let us suppose that each A(n, t), has a *state* – denoted by an opportune set of variables – and also that it can change its state for $t \in T$, showing a micro behaviour as the movement of the state values in T.

Thus we can write $\mathbf{mb}(n, t)_{t \in T}$ for the *micro* behaviour of the agents A(n) observed in period T.

Let us also suppose that we can define $\{\mathbf{C}_{1\le n\le N}\ [\mathbf{mb}(n, t_h)]\}$ for a *combination* of those micro behaviours at time t_h, where $\mathbf{C}_{1\le n\le N}$ indicates a set of *combination operation(s),* appropriately specified (sum, product, average, min, max, etc.), of values for state variables associated with the **N** agents [4].

Moreover, we write $\mathbf{MB}(t_h) = \mathbf{F}\ \{\mathbf{C}_{1\le n\le N}\ [\mathbf{mb}(n, t_h)]\}$ to represent the *macro* behaviour of **S**(t, **N)**, defined as a *recombining function* **F** (or *macro rule*) of the combination of the micro behaviours, and $\mathbf{mb}(n, t_{h+1}) = \mathbf{f}_n\ \{\mathbf{N}_n[\mathbf{MB}(t_h)]$ to represent the micro behaviour, where $\mathbf{N}_n$ represents the *necessitating operation(s)* that link(s) the micro behaviours to the macro behaviour (or the micro and macro effects).

The combinatory system, observed on a discrete time scale, can be represented as follows (figure 1) [5]:

$$
\textbf{[A]}\quad \begin{cases} \mathbf{mb}(n, t_0) \leftarrow \text{“CHANCE”} & 1\le n\le \mathbf{N} \qquad [\text{A}.1] \\ \mathbf{MB}(t_h) = \mathbf{F}\ \{\mathbf{C}_{1\le n\le N}\ [\mathbf{mb}(n, t_h)]\} & h = 0, 1, 2, \ldots \qquad [\text{A}.2] \\ \mathbf{mb}(n, t_{h+1}) = \mathbf{f}_n\ \{\mathbf{N}_n[\mathbf{MB}(t_h)]\} & 1\le n\le \mathbf{N} \qquad [\text{A}.3] \end{cases}
$$

Equation [A.1] shows that the first input is considered to be the product of *chance.*

In equation [A.2] I have indicated the same time reference (t_h), since usually the macro behaviour is *contemporaneous* to the micro behaviours, as it is derived from these.

Equation [A.3] instead describes how the subsequent *micro behaviour* $\mathbf{mb}(n, t_{h+1})$ depends on the past macro behaviour (again referring to t_h), according to a *necessitating function* $\mathbf{f}_n$ (or *micro rule*) that we assume is specified for every A(n, t) and according to the *necessitating operation(s)* represented by N_n**.**

1.3. The central idea of Combinatory System Theory (CST)

The *central idea* is that we can view a collectivity as a *combinatory system* only if the behaviour of agents is not exclusively determined by *general* rules [6] (as in the *cybernetic* approach, in *evolutionary cybernetics* [7], in *population dynamics* [8], in *systems dynamics* [9], in *Haken's synergetics* [10] *and in* the *autopoietic* approach [11]) or by *local rules* [12] (as in the traditional *complex systems approach* [13] and its related specific topics: *adaptive complex systems* [14]*, cellular automata* [15]*, Alife* [16] approaches, such as *Ants* [17], *Swarm* [18] and *Floys* [19] and so on, the *recursive* approach[20], such as *fuzzy systems* [21] and *genetic algorithms* [22]) but above all by a general *micro-macro feedback* rule, so that we *must observe*, or *assume*, mutual interdependence: the micro behaviours produce the macro behaviour, but this influences the micro behaviours in a micro-macro feedback which acts over many cycles [23].

The combinatory systems approach is *neither* a *macro* approach, since it also refers to local rules considering micro bahaviours, *nor* a *micro* approach, since it also includes the macro behaviour in the model of the *system.*

It is rather a *micro-macro* approach, precisely in that the *operating rules,* describing the behaviour of the system, must in some way include not only *local rules* but also the *feedback* between the micro and macro behaviours [24].

1.4 The three main characteristics of CST

The *interdependence* between micro and macro *rules* and *effects* constitutes the *first characteristic* of Combinatory Systems.

Recognizing the existence of a micro-macro feedback is indispensable for interpreting collective phenomena as deriving from a combinatory system: the state of the system at a given time *must* depend on the state of its agents; but this in turn *must* depend on the state of the system.

The micro-macro feedback generates a synergetic effect that produces *self-organization* and *emerging macro behaviours* which are only attributable to the collectivity.

We can thus say that in combinatory systems the micro behaviours create a *pattern in the collectivity* – normally invisible to the agents – and this pattern influences or determines the micro behaviour of the agents[25].

The *macro behaviour* – or its macro effects – may be thought of as a *dynamic attractor* toward which the micro behaviour tends and modifies over time[26]. For this reason we cannot consider in general the ants, the swarm and, more generally, the cellular automata approaches as examples of combinatory systems, except in the case in which the macro effect may affect the micro behaviours of the agents in some way [27].

We must nevertheless also recognize that each agent is normally blind to the macro behaviour of the system while being aware of the micro behaviours of some other agents; from this we immediately see a *second characteristic* of the combinatory systems: they are *incomplete* and *limited* information systems:

– they are *incomplete information systems* in that each of the $A(n) \in \mathbf{S}(t, \mathbf{N})$ produce their own micro behaviours without considering the macro behaviour of the unitary system as information (except as an extreme case of a completely observable macro effect);

– they are *limited information systems* in that the micro behaviour of A(n) depends on information about the micro behaviours (which occur or is only expected or foreseen) of a limited number of other *neighbourhoods* of A(n) (defined in an opportune way), exactly as in a cellular automaton.

The second characteristic is not in contrast with the first; they simply derive from different points of view:

– from an *external* point of view, the observer must recognize the macro behaviour and the micro-macro feedback action in order to define and build a model of the combinatory system;

– from an *internal* point of view the agents normally are unaware of the macro behaviour and act according to limited information.

In many cases, however, this second characteristic seems to fail because we can observe agents acting according to some general pattern related to the system. There is no contradiction: we must simply distinguish between the micro and macro *behaviours* and micro and macro *effects* in the environment.

When this occurs, the micro behaviours of the agents are related to some observable macro effects, and the micro and macro feedback operates between the *micro* behaviour and the *macro* effects.

The combinatory systems generally are set off "by chance", but when activated they maintain their dynamics "by necessity", due to the presence of *necessitating* and *recombining factors*.

The action of the *micro-macro feedback*, which is guaranteed by the *necessitating* and *recombining factors*, turns these collectivities into true systems which can be observed as a unit as well as a multiplicity of agents, and which I have termed *combinatory systems.*

Moreover*, to interpret the activity of combinatory systems we need always to understand the nature of both the recombining factors and the necessitating ones* since, without the joint action of these factors, there would be no micro-macro feedback and the collective phenomena the theory tries to explain would not be produced.

Often such *necessitating factors* result from obligation, convenience, utility, desire, or the operative programme of the individual agents.The agents can be aware of these (I want to adjust my step to the marching step of my companions) or not (I don't want to transmit the flu virus, but this takes place without my being aware of it).

While the *recombining* factor characterizes the macro rules, the *necessitating* factor characterizes the micro ones.

In order to explain the activity of combinatory systems we must understand the nature of the macro rules, which specify the recombining factor, and of the macro rules, which specify the necessitating factor; the joint action of these factors gives rise to and maintains the macro and micro behaviours.

The activity of the combinatory systems is thus produced by the joint action of "*chance*" and "*necessity*"; they can therefore also be called "*chance-necessity*" systems and, as such, are distinguished from operative systems, which are usually "cause-effect" systems [28].

Other relevant characteristics (I will only mention these) concern the fact that, even though combinatory systems are unorganized and closed systems, they can organize themselves into specialized subsystems and show ramifications [29] and can expand[30] their effects to agents belonging to a vaster environment.

1.5 Irreversible and reversible systems. Path dependence and chaos

If a probability is associated to the transition of state of each agent, then the combinatory system is *stochastic*; the macro behaviour depends on the probabilistic micro behaviours. In the opposite case it is *deterministic*.

In *probabilistic combinatory systems* the micro behaviour depends on a *probability of transition of state*, and is carried out in a *period of transition* of state.

Both *probabilities* and *periods* of transition of state nevertheless depend on the state of the system, so that the *micro behaviours* are in turn conditioned by the *macro behaviour* of the entire system.

The *probability of transition* should offer numerical information on all the characteristics observable, or even imaginable, in the agent A(n, t), such as to make a change of state possible, plausible, probable, likely. This thus expresses the influx of *necessitating factors* that impose on A(n, t) its own micro behaviour. In other words, it should express the likelihood of a given micro behaviour and a given micro effect which can potentially be carried out and obtained from A(n, t).

Due to the existence of the micro-macro feedback, if the state of the system derives from the state of its agents, this nevertheless influences the micro behaviours and the states of the agents in the base according to the probability of transition for each one; a probability that depends, in turn, on the state of the system.

We must therefore take account of this feedback, for example by writing that:

1) the state of each agent depends on the probability that characterizes it; but this probability is in turn a function of the state of the system;

2) the length of the period of transition of state of each agent that is modified is also a function of the state of the system.

The combinatory systems which are most interesting and easiest to represent are the *irreversible* ones where both the micro and macro behaviour produce permanent effects (residential or industrial settlements, the maintenance of the language, the spread of epidemics). Irreversible systems explain almost all the cases of *path dependence,* as we can see from [A.1] and [B.1].

In regard to combinatory system theory, recognizing the phenomenon of *path dependence* is not a theory but simply the observation that the dynamic of a social system – its macro behaviour or its macro effect – can be thought to depend on initial chance (dependence on initial conditions) and on the *recombining* rules of the micro behaviours of the agents[31].

Thus, the individual choices of the agents lead to micro behaviours deriving from past history, that is from the macro behaviour (history dependence).

In this sense the *path dependence* is the proof of the action of the micro-macro feedback, even if path dependence theory does not include this mechanism in the explanation of the path dependence[32].

The deterministic action of path dependence, the necessity, is not a consequence of the past evolution of the path of the system but of the micro-macro feedback, and thus of the necessitating and recombining factors.

Ignoring micro macro feedback leads to a second consequence: *path dependence* theory focuses particularly on the micro behaviour, considering the macro behaviour as a constraint to the individual freedom to decide.

The theory also considers reversible systems that have a cyclical behaviour and, under certain conditions concerning the probability function regarding the transition of state of the agents, a chaotic one as well.

1.6 First example – Chaotic probabilistic reversible system

As an example, consider the case of a non-ordered system where every A(n, t) is a Bernoulli random variable that, at any $t \in T$, shows only two states: **mb**(n, t) = [1 or

0].

The macro behaviour is $\mathbf{MB}(t) = N(t)$, $0 \leq N(t) \leq \mathbf{N}$, since for [A.2] we have simply established that $\mathbf{R}_{1\leq n\leq N}[\mathbf{mb}(n, t)],= \sum_{1\leq n\leq N} [\mathbf{mb}(n, t)]$.

We also suppose that the *probabilities* of transition from state "0" to state "1", p(n, N), are defined for each A(n, t) and for each $0 \leq N(t) \leq \mathbf{N}$, as well as the probability q(n, N) = 1 - p(n, N); to simplify, these probabilities might be assumed to be the same for each agent, so that we can write p(N) = 1- q(N).

We assume there is a *feedback between the micro and macro behaviour*, in the sense that the state of each agent depends on the probability p(n, N), which in turn depends on the state of the system, N(t), which defines the macro behaviour.

Let us simply assume that the function p(n, N) takes on the following values:

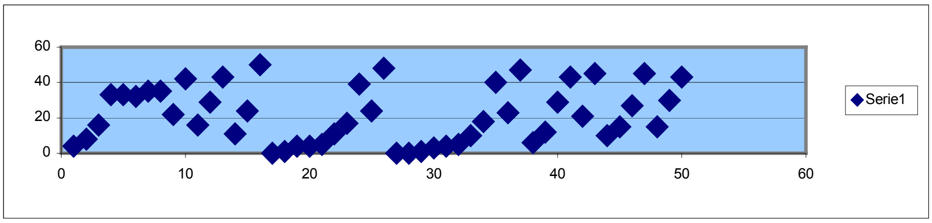
$p(n, N) = p(N) = 2(N/\mathbf{N})$ if $0<N\leq \mathbf{N/2}$

$p(n, N) = p(N) = 1-[(2N-\mathbf{N})/\mathbf{N}]$ if $\mathbf{N/2}<N\leq \mathbf{N}$.

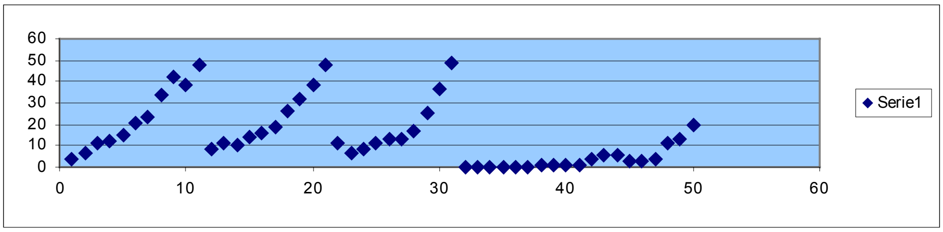
If we simulate the micro behaviour by some experiment that generates *random numbers* for each agent, we observe that after the random initial impulse that shapes mb(n, t_0), the combinatory system presents a chaotic macro behaviour **MB**(t) = N(t) (figure 2).

Fig. 2 – Reversible probabilistic combinatory system with chaotic macro behaviour

Test n. 1: **N** = 50 and N(1) = 4 due to initial *chance*



Test n. 2: the function p(n, N) increases straight line and assumes the value 1 for N(t) = (4/5 **N**), and then decreases straight line to 0



1.7. Second example – The Murmur and Noise combinatory system

Let us consider, for example, the phenomenon of a murmur arising in a crowded room. Where does it come from? From the voice levels of those present in the room who are speaking to each other. But why do they speak in a loud voice? Because there is the murmur. If the murmur increases, those present, in order to make themselves heard, must raise their voices. But this only increases the murmur, which forces those present to raise their voices even more, which increases the murmur and forces those present to.... etc., etc. The murmur is formed by the voice levels of those talking, but this in turn depends on the murmur.

We can represent this system by the model (see model **[A]** and figure 3)[33]:

$$\textbf{[C]} \quad \begin{cases} \mathbf{v}(n, t_0) \leftarrow \text{“CHANCE”} \\ \mathbf{M}(t_h) = \{\, \mathbf{k}\ (1/\mathbf{N}) \sum_{1\le n\le \mathbf{N}} [\mathbf{v}(n, t_h)] + \mathbf{Q}\ r_p(t) \,\}\ (1 - \mathbf{a}) \\ \mathbf{v}(n, t_{h+1}) = [w(n)\ \mathbf{M}(t_h) + (v_{min}(n) + v_{rnd}(n)\ l_p(n, t_h))\ ]\ s_p(n)\ b_R(n, t_h) \end{cases}$$

in which:

– $\mathbf{v}(n, t_h)$ [= $\mathbf{mb}(n, t_h)$] is the **v**oice level of $A(n, t_h)$,

– $\mathbf{M}(t_h)$ [= $\mathbf{MB}(t_h)$] is the **M**urmur arising from the group of talking people,

– $\mathbf{C}_{1\le n\le \mathbf{N}}$ [$\mathbf{mb}(n, t_h)$] = $(1/\mathbf{N}) \sum_{1\le n\le \mathbf{N}} [\mathbf{v}(n, t_h)]$ is the form of *combination* of the **N** voice levels that considers simply the *mean value* of the voice levels of talking people.

– **k** represents an *environmental noise coefficien*t,

– [$\mathbf{Q}\ r_p(t)$] represents *random noise* influencing $\mathbf{M}(t_h)$ according to the probabilities of *external random events* $r_p(t)$,

– $\mathbf{a} \le 1$ indicates the *sound-absorbing coefficient* of the environment,

– [w(n) $\mathbf{M}(t_h)$ + $\mathbf{v}_{min}$ (n)] [= $\mathbf{N}_n\mathbf{MB}(t_h)$] indicates the *necessitating functions*, where

– w(n) > 0 represents a *subjective weight*,

– $\mathbf{v}_{min}$ (n) denotes the *voice level above the background noise* that is necessary to be heard, which depends on the amount of information the speaker has (number of persons who must be reached by his voice),

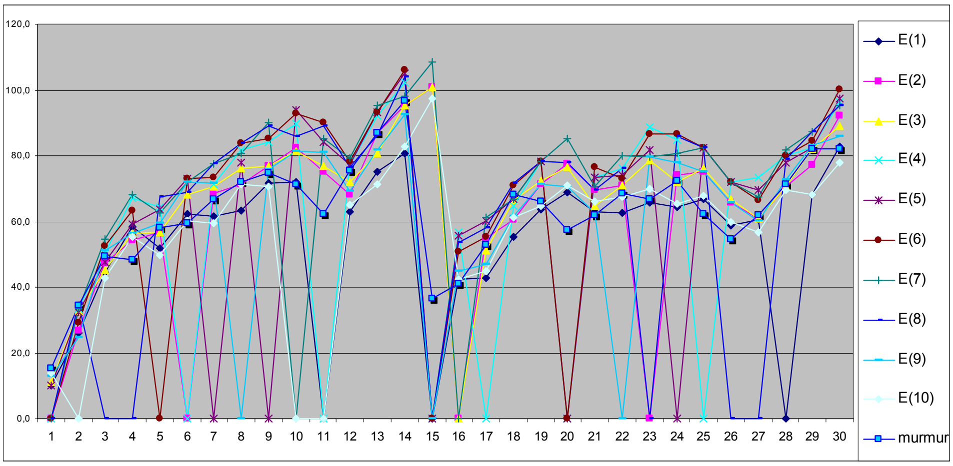
– $\mathbf{v}_{rnd}$(n) represents a *random voice level* which may influence the voice level of each speaker,

– l_p(n, t_h) denotes the *probability* of this random factor,

– s_p(n) synthesizes the probability of speaking for the A(n) (depending on education, number of talking people, attention, interest and so on, but not depending on time),

– b_R(n, t) denotes the bearing factor; if we indicate with b* the tolerance, that is the maximun level of bearing, then b_R(n, t) = 1 if $\mathbf{v}$(n, t_h) < b(n)* and b_R(n, t)= 0 if $\mathbf{v}$(n, t_h) = b* [34].

Fig. 3 - Model of Murmur and Noise system with 10 agents



1.8 Typology of combinatory systems

Combinatory systems can be ordered and classified into five classes according to the macro effect produced:

1 - systems of *ACCUMULATION,* whose macro behaviour leads to a macro effect which is perceived as the accumulation of objects, behaviours, or effects of some kind; this logic applies to quite a diverse range of phenomena, among which the formation of urban or industrial settlements of the same kind and of industrial districts, the accumulation of garbage, graffiti, writings on walls; but it can also be applied to phenomena such as the breaking out of applause, the formation of lines in fashion shows, the grouping of stores of the same type in the same street.

2 - systems of *DIFFUSION*, whose macro effect is the diffusion of a trait or particularity, or of a "state", from a limited number to a higher number of agents of the system; systems of diffusion explain quite a diverse range of phenomena: from the spread of a fashion to that of epidemics and drugs; from the appearance of monuments of the same type in the same place (the towers of Pavia, for example) to the spread and maintenance of a mother tongue, or of customs.

3 - systems of *PURSUIT*, which produce a behaviour that consists in a gradual shifting of the system toward an objective, as if the system, as a single entity, were pursuing a goal or trying to move toward increasingly more advanced states; this model can represent quite a different array of combinatory systems: from the pursuit of records of all kinds to the formation of a buzzing in crowded locales; from the start of feuds and tribal wars in all ages to the overcoming of various types of limits.

4 - systems of *ORDER*, which produce a macro behaviour, or a macro effect, perceived as the attainment and maintenance of an ordered arrangement among the agents that form the system; systems of order can be used to interpret a large number of phenomena: from the spontaneous formation of ordered dynamics (for an observer) in crowded places (dance halls, pools, city streets, etc.) to that of groups that proceed in a united manner (herds in flight, flocks of birds, crowds, etc.); from the creation of paths in fields, of wheel-ruts on paved roads, of successions of holes in unpaved roads, to the ordered, and often artificial, arrangement of individuals (stadium wave, Can-Can dancers, Macedonian phalanx).

5 - systems of *IMPROVEMENT AND PROGRESS*, whose effect is to produce progress, understood as an improvement in the overall state of a collectivity [35]. There are three fundamental types of systems of improvement and progress:

a) <u>Medial systems</u>

b) Maximal systems, or systems of pursuit

c) Minimal systems, or systems of flight.

These will be presented in SECTION 2.

The four most evident *systems of improvement and progress* which every business operates in are:

1) Increasing productivity
2) Greater quality
3) From Needs to Aspirations
4) Scientific and technical progress

These will be summarized in SECTION 3.

2 - Systems of improvement and progress

2.1 The general definition

We indicate by **S**(t, **N**) = [t, A(1, t), ..., A(n, t), ... , A(**N**, t)], a combinatory system (with or without a shape) composed of a base of **N** agents, A(n, t), $1 \le n \le \mathbf{N} \ge 2$, and we assume:

- that every A(n, t) $\in$ **S**(t, **N**) is characterized – with regard to the *micro behaviour*, or the *micro effect* – by a variable $\mu(n, t)$ (or a vector of variables) that represents the state of the agent A(n) at time "t" and the *micro behaviour* $\mathbf{mb}(n, t)_{t \in T} = \mu(n, t)$, in T. Such a $\mu(n, t)$ can be defined as an *improvement variable* (or index, or measure, or parameter) of the behaviour (or in the states) of that agent, according to value parameters to be defined from observation or stipulation, if $\mu(n, t_2) > \mu(n, t_1)$, for every $t_2 > t_1$ of T (or the opposite inequality, according to the meaning of $\mu(n, t)$);
- that the entire **S**(t, **N**) is characterized—in its *macro behaviour,* or *macro effect* – by a variable (or a vector of variables [36]) $\pi(S, t) = \mathbf{C}_{1 \le n \le N}\, [\mu(n, t)]$, that represents an index (or measure, or parameter) of *progress* in the behaviour (or in the states) of the system itself. In particular, there is *progress* in the system – according to

value parameters to be defined by observation – if $\pi(S, t_2) > \pi(S, t_1)$, for every $t_2 > t_1$, (or the opposite inequality, according to the meaning of $\pi(S, t)$).

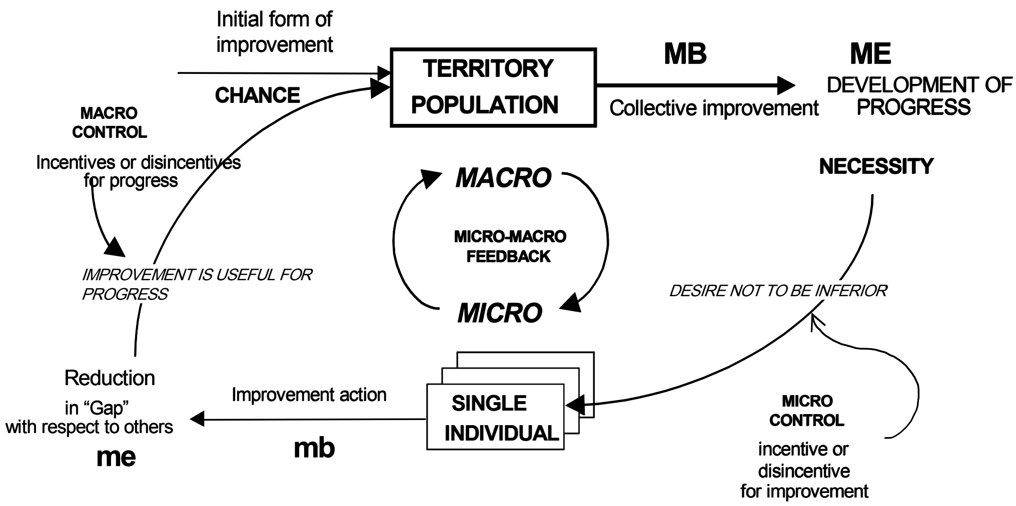
According to model **[A],** presented in a previous section, we can formalize the following general model of a *combinatory system* showing *"improvement" and "progress"*:

$$
\textbf{[E]} \quad \begin{cases} \mathbf{mb(}n, t_0) = \mu(n, t_0) \leftarrow \text{"CHANCE"} & 1 \leq n \leq \mathbf{N} \quad [E.1] \\ \mathbf{MB(}t) = \pi(S, t) = \mathbf{F}\ \{\mathbf{C}_{1\leq n\leq \mathbf{N}}\ [\mu(n, t)]\}, & t \in T, \ldots \quad [E.2] \\ \mathbf{mb(}n, t{+}dt) = \mu(n, t{+}dt) = \mathbf{f}_n\ \{\mathbf{N}_n[\pi(S, t)]\} & 1 \leq n \leq \mathbf{N} \quad [E.3] \end{cases}
$$

When "by chance" an improvement begins in one or all of the agents of the system, then "by necessity" progress occurs throughout the system; the improvement spreads and the progress continues, unless a *limiting state* is reached in which no further improvement can be carried out and no further progress can occur[37].

A simple descriptive model of the systems of improvement and progress is shown in figure 4.

Fig. 4 - Model of the combinatory system of improvement and progress



2.2 Three types of systems of improvement and progress

In order to specify model **[E],** the *first* step is to define the *combining operation* $\mathbf{C}_{1\leq n\leq N}$ [μ(n, t)]; the *second* step is to define the *necessitating functions* $\mathbf{N}_n[\pi(S, t)]$ and to specify the shape of the functions **F** and **f**.

Considering the *first* step, we can observe that there are three basic forms of systems of improvement and progress which are characterized differently depending on the way in which the *improvement* parameter interacts with the *progress* parameter.

a) MEDIAL SYSTEMS: here the micro behaviours aim at reaching and/or exceeding a parameter of progress which represents an *AVERAGE* (whose form must be specified) of the measures of the parameter of improvement in the base agents; the macro behaviour of the system leads to a continual readjustment of the average, so that the individual improvement leads to an advancement in the average progress, which, in turn, gives a boost to individual improvement;

b) MAXIMAL SYSTEMS, or "inferiority-reducing" or even "of pursuit": these are characterized by the fact that the parameter of progress is represented by the *MAXIMUM* value assumed by the parameters of improvement which characterize the agents of the system (the agent to which this value belongs is referred to as "the best"); all the other agents thus present a state which is inferior to the best and try to improve for their part; the agent that succeeds in being the best becomes the *guide for progress* and gives a push toward further improvement. We thus witness micro behaviours aimed at reducing the inferiority with respect to the level of progress, and this causes a macro behaviour whose effect is to raise the average level of improvement, so that some agents manage to further raise the previous level of progress;

c) MINIMAL SYSTEMS, or "superiority-incrementing" or even "of flight": these systems act in a symmetrical way with respect to the previous ones, since the parameter of progress is represented by the *MINIMUM* level reached by the improvement parameter; all the other micro behaviours are thus superior. Each agent of the system tries to outdistance as much as possible its own level of improvement from the level of progress, to flee from the minimum level of improvement, to increment its own superiority. This leads to a general increase in the average level of improvement, which ends up raising the parameter of progress, further boosting the levels of improvement.

2.3. MEDIAL SYSTEMS of improvement and progress

Continuing with the *second* step, in order to specify model **[E]** we will consider the simplest case, that of the MEDIAL SYSTEMS, where the *progress* parameter, $\pi(S, t)$, derives from an *AVERAGE* of the measures of $\mu(n, t)$, $1 \le n \le \mathbf{N}$; to further simplify the situation, let us assume that $\pi(S, t)$ corresponds to the simple *arithmetical mean* of the levels of the parameter of improvement measured at time t, with the average itself referring to time t (every other choice regarding the time period is permissible).

The previous assumption allows us to derive the following model (figure 5):

$$
\textbf{[F]} \quad \begin{cases} \mathbf{mb}(n, t_0) = \mu(n, t_0) \leftarrow \text{“CHANCE”} & 1 \le n \le \mathbf{N} \qquad \text{[F.1]} \\ \mathbf{MB}(t) = \pi(S, t) = (1/\mathbf{N})\, \Sigma_{1 \le n \le N}\, \mu(n, t), & \forall t \in T, \ldots \qquad \text{[F.2]} \\ \mathbf{mb}(n, t+dt) = \mu(n, t+dt) = \mu(n, t+dt) = \{ \mu(n, t) + & \end{cases}
$$

$$
+ \, p_{\Delta[1,0]}(n, t)\, i_R(n, t)\, [\mu(n, t) - \pi(S, t)] \} +
$$

$$
+ \, \{ r_{\Delta[1,-1]}(n, t)\, [k\, \mu(n, t)] \} \qquad 1 \le n \le \mathbf{N} \qquad \text{[F.3]}
$$

in which the factor

$$
p_{\Delta[1,0]}(n, t)\, i_R(n, t)\, \Delta\mu(n, t),
$$

for each A(n), represents the *necessitating functions* $\mathbf{N}_n[\pi(S, t)]$ in the sense that:

a) $\Delta\mu(n, t) = \mu(n, t) - \pi(S, t)$ at time t denotes the *deviation* between the individual improvement level and mean level denoting progress; so that each A(n) perceives an *inferiority*, with respect to the mean, if $\Delta\mu(n, t) < 0$, or a *superiority* in the opposite case;

b) $p_{\Delta[1,0]}(n, t)$ represents the *necessitating factor* (a weight, a probability, an intensity, etc.) – which varies for each A(n), depending on the *internal random events* of A(n) (ability, need, will, decision, resolution, strength or weakness, shyness, information, and so on), and at each moment – according to which each A(n) takes account of the deviation in order to readjust the parameter of improvement. In other words, $p_{\Delta[1,0]}(n, t)$ states "if" A(n) may readjust $\mu(n, t)$; the foot symbol Δ means that $p_{\Delta[1,0]}(n, t)$ may be different for the cases of $\Delta\mu(n, t) \ge 0$ or for the opposite case; the foot symbols [0, 1] mean that $p_{\Delta[1,0]}$ admits only the possibility of positive improvement with respect to the progress, not that of negative improvement; in other words, $p_{\Delta[1,0]}$, for each agent, states the probability of reducing the negative gap or maintaining the positive gap and not the possibility of worsening the position with respect to the progress (of course this restriction may be relaxed);

c) $i_R(n, t)$ represents a random coefficient of improvement that states "how much" A(n) may readjust $\mu(n, t)$; so that A(n) may or may not improve its position, according to the probability $p_{\Delta[1,0]}$, and in the "yes" case the measure of improvement is determined by $i_R(n, t)$;

The factor:

$$\{ r_{\Delta[1,-1]}(n, t) [k \mu(n, t)] \},$$

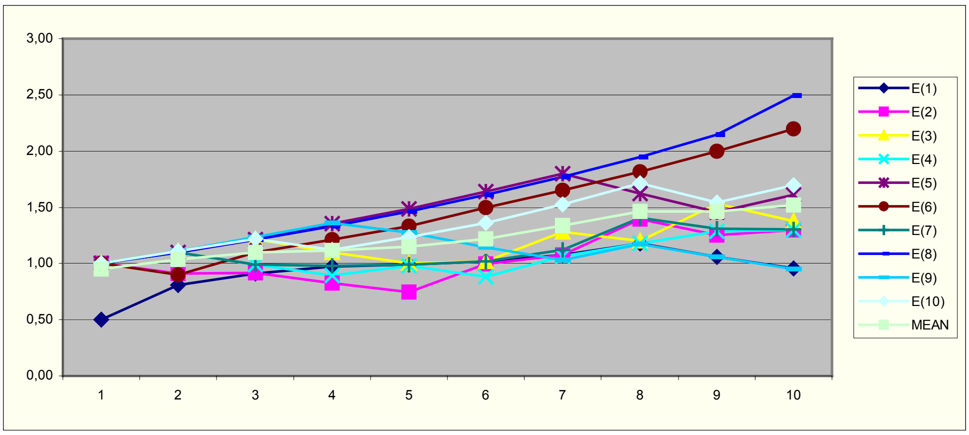
for each A(n), represents a *contingent factor*, according to the coefficient k, in the sense that:

a) $r_{\Delta[1,-1]}(n, t)$ indicates the *probability* that the n-th agent in period t will undergo a *random* variation in its own level of improvement;

b) the foot symbols [1,-1] mean that $r_{\Delta[1,-1]}(n, t)$ may represent an *external favourable* (if 1) *or unfavourable* (if -1) *random event* (luck, competitors, ignorance and so on). If the favourable external event occurs compatible with the probability $r_{\Delta[1,-1]}(n, t)$, then the parameter of improvement of A(n, t) is affected by $[k \mu(n, t)]$; otherwise by $-[k \mu(n, t)]$;

c) the foot symbol Δ means that $r_{\Delta[1,-1]}(n, t)$ may also be different for the cases of $\Delta\mu(n, t) \geq 0$ or for the opposite case.

Fig. 5 - Model of a medial system with ten agents



2.4 MAXIMAL SYSTEMS (of pursuit) of improvement and progress

The operating logic of the MAXIMAL SYSTEMS is similar to that of the MEDIAL SYSTEMS. From model [F] we arrive at the following:

$$\textbf{[G]}\quad \begin{cases} \mathbf{mb}(n, t_0) = \mu(n, t_0) \leftarrow \text{“CHANCE”} & 1 \le n \le \mathbf{N} \quad \text{[G.1]} \\ \mathbf{MB}(t) = \pi(S^M, t) = \mathrm{Max}_n\, \mu(n, t) = \mu(n^M, t), & \forall t \in T, \quad \text{[G.2]} \\ \mathbf{mb}(n, t+dt) = \mu(n, t+dt) = \mu(n, t+dt) = \{\, \mu(n, t) + \\ \quad + p_{\Delta[1,0]}(n, t)\, i_R(n, t)\, [\mu(n, t) - \pi(S^M, t)] \,\} + \\ \quad + \{\, r_{\Delta[1,-1]}(n, t)\, [k\, \mu(n, t) + h\, \pi(S, t)] \,\} & 1 \le n \le \mathbf{N} \quad \text{[G.3]} \end{cases}$$

From equation [G.2] we observe that MAXIMAL SYSTEMS differ in that the *parameter of progress* is represented by the *maximum* level of the parameter of improvement [$\mathrm{Max}_n\, \mu(n, t)$] reached by the base agents.

The agent $A(n^M)$ which provides the maximum for $\mu(n^M, t)$ can be called the *leader agent*, since its performance represents the degree of progress for the entire system.

The term $\Delta\mu(n, t) = \mu(n, t) - \pi(S^M, t) = \mu(n, t) - \mathrm{Max}_n\, \mu(n, t)$ represents the *quantum of inferiority*; thus it can never take on positive values, and will be equal to zero for the *leader agent* (assuming for simplicity's sake - though it is not necessarily the case - that this agent is unique);

The level of the parameter of *improvement* at time (t+dt) depends in this case as well on the level of the parameter of *progress,* for example according to [G.3], which is entirely similar to [F.3], except for the term $\{\, r_{\Delta[1,-1]}(n, t)\, [k\, \mu(n, t) + h\, \pi(S, t)] \,\}$.

At time $t<t_0$ we have $\mu(n, t) = 0$ for each $1 \le n \le \mathbf{N}$. At time t_0 we can instead assume that, "by chance", $\mu(n, t_0) > 0$ for some n. We identify the "*leader*" *agent* in terms of improvement - that is, the agent $A(n^M)$ - and we use it as the indicator of progress. At this point the combinatory system begins to operate as a system of pursuit, since the other agents "pursue" the leader and try to eliminate the gap between their performance and that of the "best"; we thus can observe a rise, "by necessity", in both the levels of *improvement* and *progress*.

The system, however, gives rise to a *second type of progress*, which takes place even when the parameter of progress determined by [G.2] does not undergo an increase; this progress consists in *a rise in the average level of the measures of improvement* for the agents in the system. In fact, it is easy to see – observing the expression $\{\, r_{\Delta[1,-1]}(n, t)\, [k\, \mu(n, t) + h\, \pi(S, t)] \,\}$ – that the attempts to reach and overtake the *leader* can push the agents to raise their individual performance; even if the level of improvement does not change, the system increases the average level of individual improvement.

Naturally "chance" can influence the dynamics of the levels of improvement, and thus that of progress, since it can reduce or accentuate the attempt of individuals - as well as the leader - to raise their level of individual improvement.

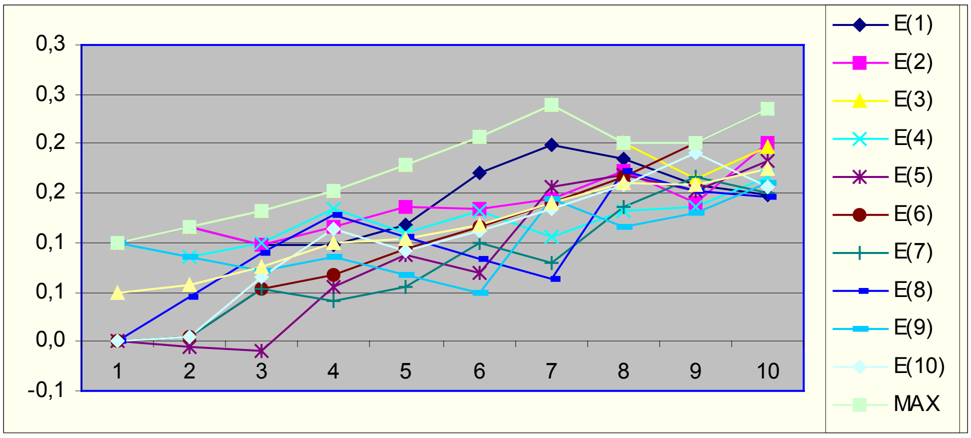
Due to the effect of chance, some other agent may also substitute the leader in terms of improvement.

Figure 6 represents the dynamics of the indices μ(n, t) for each (t) for a maximal system of ten agents.

The typical MAXIMAL SYSTEM is the "record" system. Each breaking of the record leads to a pursuit of the record, with an improvement in the average level of performance.

The "growing productivity" and "improving quality" systems are also maximal systems. Every technological innovation provides an advantage to the company that introduces it in terms of productivity or quality; the competing companies must, if they do not want to reduce their profits, necessarily follow the leading company by trying to overtake it. The latter tries again to maintain its leadership, and thus the typical maximal system is triggered.

Fig. 6 - Model of a ten-agent maximal system



2.5 MINIMAL SYSTEMS (of flight) of improvement and progress

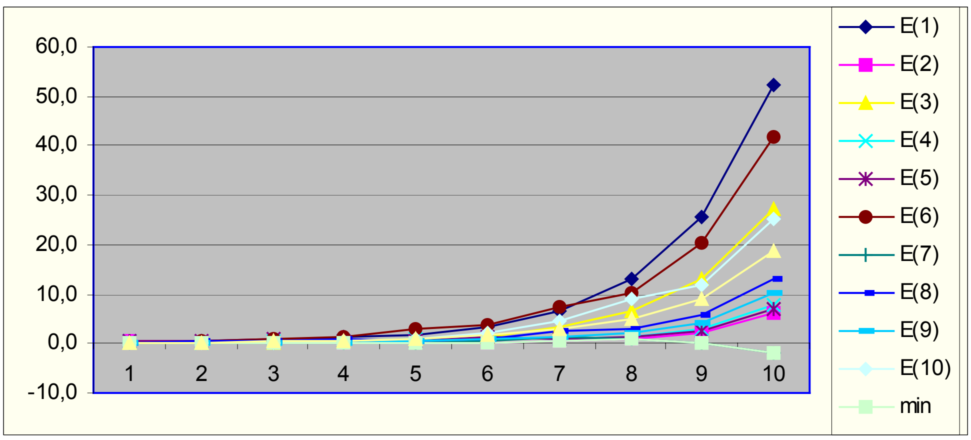
MINIMAL SYSTEMS operate to achieve improvement and progress according to a logic that is the opposite of that for maximal systems, since the parameter of *progress* for the entire system is represented by the *minimum level* of the parameter of *improvement* reached by the base agents, as shown in model [**H**].

$$[\mathbf{H}]\quad \begin{cases} \mathbf{mb}(n, t_0) = \mu(n, t_0) \leftarrow \text{“CHANCE”} & 1 \le n \le \mathbf{N} & [\text{H.1}] \\ \mathbf{MB}(t) = \pi(S_m, t) = \text{Min}_n\, \mu(n, t) = \mu(n_m, t) & \forall t \in T, \ldots & [\text{H.2}] \\ \mathbf{mb}(n, t+dt) = \mu(n, t+dt) = \mu(n, t+dt) = \{\, \mu(n, t) + & & \\ \quad + p_{\Delta[1,0]}(n, t)\, i_R(n, t)\, [\mu(n, t) - \pi(S_m, t)] + & & \\ \quad + \{\, r_{\Delta[1,-1]}(n, t)\, [k\, \mu(n, t) + h\, \pi(S, t)] \,\} & 1 \le n \le \mathbf{N} & [\text{H.3}] \end{cases}$$

The agent $A(n_m)$ that provides a minimum value for $\mu(n_m, t)$ can be called the *base agent*, since it is the basis for the flight of the other agents, who tend to increase as much as possible the distance between the level of improvement they have achieved and the minimal improvement of the base agent.

In this system, the relation $\Delta\mu(n, t) = \mu(n, t) - \pi(S_m, t) = \mu(n, t) - \text{Min}_n\, \mu(n, t)$ represents the *quantum of superiority*; thus, it can never take on negative values and will be equal to zero for the *base agent* (assuming, for simplicity's sake, though not necessarily, that this is a unique agent).

Fig. 7 - Model of a ten-agent minimal system



At time $t<t_0$ we can have $\mu(n, t) = 0$ for each $1\leq n\leq \mathbf{N}$. At time t_0 we can instead assume that, "by chance" $\mu(n, t_0) > 0$ for some n. We identify the base improvement agent (that is, the agent $A(n_m)$) and we take it as the progress indicator. At this point the combinatory system begins to operate as a system of flight, since the other agents "flee" from the base, trying to increase the gap between their performance and that of the "worst" agent; we thus witness the increase "by necessity" in both the improvement and progress levels.

However, in this form as well the system brings about a second type of progress that is revealed even when the progress parameter identified by [H.2] is not increased, and which consists in the *raising of the average* level of the improvement measures of the system's agents.

Here, too, "chance" can influence the dynamics of the improvement levels, and thus the progress levels, since it can reduce or accentuate the attempts of the individuals to raise their individual improvement levels.

Figure 7 shows the dynamics of the indices $\mu(n, t)$ for a minimal system of ten agents

We can consider as minimal the systems of scientific and technological innovation. Every scientific discovery or invention is received by the other researchers, who use it as the basis for new research, thereby raising the average level of information and research. The researchers must further improve their research by producing new progress in their fields.

2.6. Conclusion. The heuristic value of Combinatory Systems Theory

After having presented the theory of combinatory systems the question arises as to why this theory, even though it is based on simple features, is able to explain so many and so varied a number of phenomena.

To answer this we must recall that there are at least three techniques for explaining a phenomenon:

a) *the classical explanation*, or cause and effect explanation. According to this technique, normally used in the experimental sciences, a phenomenon is explained by making it derive from others, which are considered to be its causes; the explanation is enriched by introducing scientific laws and theories that should justify the fact that the phenomenon to be explained depends on the causes, taking account of several particular conditions regarding the situation in which the observed phenomenon occurs [38];

b) *systemic explanation*. Sometimes there is no cause for a phenomenon; the latter is mutually related to other phenomena and, at the same time, affects these through a continuous feedback. The explanation thus consists in reconstructing the system which the observed phenomenon is a part of; an understanding of the systemic ties is enough to understand how the phenomenon is produced as a function of all the other phenomena;

c) *procedural explanation*; a very common type of explanation, used whenever a phenomenon does not derive from others that produce it (causes) or are interrelated (system), but is instead the result of the application of some procedure (process, programme, formulation, algorithm, etc.) that allows the phenomenon to be obtained as the consequence of given, or assumed, rules.

Darwin's theory of evolution, for example, represents a powerful procedural explanatory tool for phenomena connected with the evolution of species. It is clear that even the theory of combinatory systems represents an efficient tool of *system thinking* for the procedural explanation of dynamic phenomena that derive from the action of collectivities that can be considered as observational units and not only as aggregates of individuals. This theory explains how the behaviour of that unit arises and evolves, by examining the interactive mechanisms between individual behaviours (micro) and collective ones (macro) and by trying to determine the *rules* that give rise to such behaviour (and their effects). Three aspects of this theory make it particularly effective:

1 - it is not limited to describing the macro behaviour of the unit based on general rules or the individual behaviours based only on local rules, but tries to uncover and explain above all the *feedback* between the macro and micro behaviours or their effects;

2 - to understand the phenomena attributable to the action of combinatory systems the theory tries to uncover and make clear the *necessitating factors* (that cause the micro behaviour of each agent in the system) and the *recombining factors* (that produce and maintain the unit's macro behaviour). The theory then concludes that, in the presence of suitable necessitating and recombining factors, «*chance*» will trigger the dynamic process of the system that «*by necessity*» is then maintained and influences the individual behaviours;

3 - the procedural explanation offered by the theory not only allows us to understand the operating mechanism that produces the phenomena under examination, but also permits us to determine the most effective forms of control.

3 – Four Systems of improvement and progress

3.1 Increasing productivity

The heuristic model can take on the following form:

MICRO RULE = NECESSITATING FACTOR - If your unit profit falls and you want to remain in the economic system as a producer - and if you cannot alter the selling price - you must reduce the average unit cost of production and increase productivity to the same level - or higher - as the average level of the other producers you are competing with, by searching for some productivity factor;

MACRO RULE = RECOMBINING FACTOR - The introduction of a productivity factor improves the average level of productivity in the system, thereby eliminating the advantages for the producer; the producers try to equal - or preferably exceed - the average level of productivity in the system;

MICRO-MACRO FEEDBACK = CHANCE AND NECESSITY - The increase in the average level of productivity in the system is the result of past micro behaviours, but this also conditions the search by the individual producers for new factors of productivity.

3.2 Greater quality

The heuristic model can take on the following form:

MICRO RULE = NECESSITATING FACTOR - If your sales fall with respect to those of other producers who have introduced quality improvements in their products or processes, and you want to remain in the economic system as a producer - and if you cannot influence your productivity by reducing your marginal unit costs - you must in turn try to improve the quality of your products;

MACRO RULE = RECOMBINING FACTOR - An improvement in the quality of a product raises the average level of quality of similar products in the productive system; the individual quality of each producer must at least equal - or preferably exceed - the average level of quality in the system;

MICRO-MACRO FEEDBACK = CHANCE AND NECESSITY - The improvement in the average quality in a productive system is the result of past micro behaviours, but this itself conditions the search by individual producers for new qualitative improvements. An innovation that produces an increase in quality is introduced "by chance"; but the innovation to which the producer turns has a negative effect on sales, and thus on the economic benefits of the other producers, forcing the latter to find "by necessity" the means and forms to improve the quality of their products.

3.3 From Needs To Aspirations

The heuristic model can be formulated as follows:

MICRO RULE = NECESSITATING FACTOR - If the quality of your life is below the average level, try to improve it; in any case try to increase the level of satisfaction of your and your family's needs and aspirations;

MACRO RULE = RECOMBINING FACTOR - The increase in the quality of life of individuals leads to a rise in the overall quality of life; the search by individuals for a larger quantity of goods and of better quality leads to a quantitative and qualitative improvement in production;

MICRO-MACRO FEEDBACK = CHANCE AND NECESSITY - The improvement in the average quality of life at the environmental level is the result of past micro behaviours, but this conditions the search by the individual consumers for new improvements in the quality of life.

3.4 Scientific and technical progress

The heuristic model can be formulated as follows:

MICRO RULE = NECESSITATING FACTOR - If you are motivated toward scientific or technological research and you uncover lacunae or the need to complete and broaden the patrimony of scientific and technical culture, dedicate yourself to research and try to make new discoveries;

MACRO RULE = RECOMBINING FACTOR - The new discoveries lead to progress in the state of science and technology; if, on the one hand, the stock of scientific and technological knowledge is enriched, on the other hand knowledge gaps are revealed that move people to undertake new research;

MICRO-MACRO FEEDBACK. CHANCE AND NECESSITY - The individual results of scientific and technological research form the patrimony of knowledge, which in turn creates the need for new research.

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http://alife.org/index.php?page=alife&context=alife
http://alife.santafe.edu/
Artificial societies - http://www.generativeart.com/
Attractors - http://www.c3.lanl.gov/~rocha/ises.html.
Autopoietic systems - http://www.enolagaia.com/EA.html#S
Cellular automata - http://www.brunel.ac.uk/depts/AI/alife/al-ca.htm
Floys - http://www.aridolan.com/JavaFloys.html
Fuller's synergetics - http://www.teleport.com/~pdx4d/synhome.html
Fuzzy logic and systems:
http://cs.felk.cvut.cz/~xobitko/ga/
http://home.online.no/~bergar/mazega.htm
Game of life and applets for Game of life:

http://serendip.brynmawr.edu/complexity/life.html#conway.
http://www.csd.uwo.ca/faculty/akd/PERSONAL/hp.html
http://www.multimania.com/ldavid/indexe.html
http://bloch.ciens.ucv.ve/~felix/Java/Simulation/Conway/
http://hensel.lifepatterns.net/
L-systems - http://www.cpsc.ucalgary.ca/projects/bmv/vmm/title.html
Nonlinear spatial-temporal interactions - http://www.cs.iastate.edu/~honavar/alife.isu.html
Pattern formation - http://www.necsi.org.
PRINCIPIA CYBERNETICA WEB, http://pespmc1.vub.ac.be/BVSR.html
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http://beelab.cas.psu.edu/research/rC/studiesSO.html
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http://ais.gmd.de/~diprimio/bar/workshops/ws4/plain/BAR-Poster-fdp.html
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http://sysdyn.mit.edu/home.html
http://www.uni-klu.ac.at/users/gossimit/links/bookmksd.htm
Teams of agents - http://borneo.gmd.de/AS/art/index.html

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[1] The Italian version of the *Combinatory System Theory* was edited in 1999 by FrancoAngeli, Milan, in the book entitled *Razionalità e libertà nel comportamento collettivo*. The English version of the Theory is on the site: www.ea2000.it/cst where you can find the simulation models for the systems described in this paper. The site www.ea2000.it contains an interview (Italian version) which clarifies many particular topics of the theory.

[2] We can distinguish combinatory systems with respect to the nature of their elements: populations or social systems, composed of rational agents; flocks (and variants), composed of instinctive and reactive agents; collectivities in any case or in a more general sense.
"It appears to leave human organisations and institutions little different in principle from wasp's nests or even piles of sand. They can all be said to emerge from the actions of the individuals. The difference is that while we assume that, for instance, wasps have no ability to reason - they just go about their business and in doing so construct a nest - people do have the ability to recognise, reason about and react to human institutions, that is, to emergent features. Behaviour which takes into account such emergent features might be called `second order emergence" (Gilbert, 1995).

[3] A combinatory system is ordered , or has a form, if the agents are arranged in an orderly way in a vector or even a multidimensional matrix. Systems with a form can have an emerging macro behaviour for some observers who have specified their point of view.

[4] For simplicity's sake such variables are not explicitly included in models [A] and [B].

[5] From model **[A]**, we can write, more completely, the model:

$$
\textbf{[B]} \quad \begin{cases} \mathbf{mb}(n, t_0) \leftarrow \text{"CHANCE"} & 1 \le n \le \mathbf{N} \qquad \text{[B.1]} \\ \mathbf{MB}(t_h) = \mathbf{F}\ \{\ \mathbf{MB}(t_{h-1}),\ \mathbf{C}_{1 \le n \le \mathbf{N}}\ [\mathbf{mb}(n, t_h)\ ]\ \}, & h = 0, 1, 2, \ldots \qquad \text{[B.2]} \\ \mathbf{mb}(n, t_{h+1}) = \mathbf{f}_n \{\mathbf{N}_n[\mathbf{MB}(t_h)]\ \} & 1 \le n \le \mathbf{N} \qquad \text{[B.3]} \end{cases}
$$

Model [B] instead assumes that the *macro behaviour* is determined as well by the past history of the system. Equation [B.2] expresses the *macro behaviour* at a given instant, in part as a function of the macro behaviour of the preceding instant.
In both cases, [A.3] and [B.3] represent the fact that the micro behaviour is independent of the micro behaviours from the preceding moments.

[6] For more deta

[7] *Evolutionary cybernetics*, based on fundamental Darwinian principles, aims to develop a theory to explain the process of arranging components to form a pattern different from what could occur *by chance***.**
See PRINCIPIA CYBERNETICA WEB, http://pespmc1.vub.ac.be/BVSR.html
See also Gould (1994)

[8] The population dynamics approach aims to represent population dynamics in terms of the dynamics of the number of agents, for example using Malthusian models and Volterra-Lokte equations in various forms (Volterra, 1926).

[9] The systems dynamic approach (Forrester, 1961), connected to Systems thinking, is a method and a technique for understanding how the behaviour of concrete collectivities arise and change over time. Internal feedback loops within the structure of the system influence the entire system's behaviour.
For more, see:
http://www.albany.edu/cpr/sds/
http://sysdyn.mit.edu/home.html
http://www.uni-klu.ac.at/users/gossimit/links/bookmksd.htm

[10] Synergetics is defined as the science of co-operation, and Haken pioneered the scientific analysis of hierarchically organized co-operative phenomena in physics, with applications also in biology and the social sciences*"* (Haken, 1997).
The *synergetics approach* provides an exogenous description of complex systems without entering into internal operative mechanisms and without examining the micro and macro rules from which the behaviour originates (Serra – Zanarini, 1990; Corning, 1995).
I have specified that I am speaking of Haken's *synergetics,* to avoid confusion with the language used by R. Buckminster Fuller and E. Applewhite to construct a metaphysical description of the world.
The integration of geometry and philosophy in a single conceptual system providing a common language and accounting for both the physical and metaphysical (Kirby Umer in: http://www.teleport.com/~pdx4d/synhome.html)
[11] See: ENCYCLOPAEDIA AUTOPOIETICA WEB**;** http://www.enolagaia.com/EA.html#.
The *autopoietic* approach is based on two fundamental concepts: 1) the idea of *behavioral coupling*; 2) the idea of *operational closure* of the system (Maturana – Varela, 1980; Varela, 1979). With regard to the first idea, it is interesting to remember that the idea of behavioral coupling is related, or derived from, that of structural coupling. *"In general, when two or more plastic dynamic systems interact recursively under conditions in which their identities are maintained, the process of structural coupling takes place as a process of reciprocal selection of congruent paths of structural changes in the interacting systems, which results in the continuous selection in them of congruent dynamics of state."* (Maturana – Guiloff, 1980, p. 139). *"Phrased more succinctly, structurally-coupled systems ... will have an interlocked history of structural transformations, selecting each other's trajectories."* (Varela, 1979,. pp. 48-49).
For more, see: http://www.enolagaia.com/EA.html#S
[12] The simulation of social behaviour by local rules (Gaylord and D'Andria, 1998, pag. xvi) may be defined as a *"Synchronic analysis [that] rules out the possibility of testing an important class of explanations, those based on process. If you want to understand why a person acts as she does, it is certainly possible to look around in the immediate environment for an explanation. But often an explanation needs to look also, or perhaps primarily, at events that occurred in the past and at how the present situation developed from previous circumstances."* (Gilbert, 1994 and 1995). See also, for further details:
http://borneo.gmd.de/AS/art/index.html.
[13] The *Complex systems approach* is the new science studying the *collective* behavior of many basic but interacting units which, obeying only *local rules*, lead to macroscopic patterns (Stacey, 1995; Coveney – Highfield, 1995; Forrest – Jones, 1994). The only reasonable approach to complexity in such systems is *synthetic*: to recognize or to define the *micro rules* which produce or direct the micro behaviours See, also: http://www.necsi.org.
In other words: *not to describe a complex system with complex equations, but to let the complexity emerge from the interaction of simple individuals following simple rules*. This *bottom-up* approach is called *Agent-Based Modeling* (ABM) (Axelrod, 1997, Darley and Kauffman, 1997).
See, for more details: http://www.aridolan.com/ and http://www.swarm.org/csss-tutorial/frames.html.
"*There is no single Theory of Complexity, but several theories arising from the various sciences of complexity, such as biology, chemistry, computer simulation, evolution, mathematics and physics. The work referred to will be that undertaken over the past three decades by scientists associated with the Santa Fe Institute in New Mexico, and particularly that of Stuart Kauffman and John Holland on complex adaptive systems (CAS), as well as the work of scientists based in Europe, such as Prigogine, Sengers, Nicolis, Allen and Goodwin*." (E. Mitleton-Kelly, 1997; Gell-Mann,1995) .

[14] The term *Complex adaptive systems* is used by the Santa Fe scientists to describe complex systems which adapt through a process of self-organisation and selection (Holland, 1995; Allen, 1997, Gell-Mann, 1994).
Here, we will use the term "complex adaptive system" to refer to a system with the following properties:
– a collection of primitive components, called "agents"– interactions among agents and between agents and their environment,
– agents adapt their behaviour to other agents and environmental constraints,
– as a consequence, system behaviour evolves over time,
– unanticipated global properties often result from the interactions.
Agent may be defined as "A natural or artificial entity with sufficient behavioural plasticity to persist in its medium by responding to recurrent perturbations within that medium so as to maintain its organisation.". For a complete classification of agents see Goldspink, 2000.
See also: http://www.cs.iastate.edu/~honavar/alife.isu.html
[15] The *cellular automata* approach, which may be considered the most general approach to simulate behaviours in collectivities (Schatten, 1999). A set of rules defines the transition from one state to another from one step in the time frame. It is important to note that the rules that define the micro behaviour of a cell are only *local rules,* in the sense that the state of the cell depends only on one of a specified number of neighbours and not on the state of the array. From the theoretical point of view, *Cellular Automata* (CA) were introduced in the late 1940s by von Neumann – A. W. Burks, 1966.
For details, see: http://www.brunel.ac.uk/depts/AI/alife/al-ca.htm.
See, for more: Gardner, 1970; Dewdney, 1989; Dewdney, 1990 (http://www.csd.uwo.ca/faculty/akd/PERSONAL/hp.html). Toffoli – Margolus, 1987 and Ulam, 1986, 1991.
One of the most famous examples of the application of the cellular automata approach is the Game of Life, invented by John Horton Conway and described by Gardner, 1970.
For more see: http://serendip.brynmawr.edu/complexity/life.html - conway.
In the web we can find many applets (Java) to easily play the game; for instance:
http://www.multimania.com/ldavid/indexe.html
http://bloch.ciens.ucv.ve/~felix/Java/Simulation/Conway/
http://hensel.lifepatterns.net/
[16] The Alife approach refers to cells simulating simple living autonomous reactive agents to show how interactions among neighboring agents, following local rules, lead, at a higher level, to complex patterns of self-organization (Coveney and Highfield, 1995).
See, for more details: http://alife.org/index.php?page=alife&context=alife and http://alife.santafe.edu/
[17] Although each ant is characterised by limited capabilities (limited local movement, recognizing food or ants, marking territory with chemical traces and so on) and acts blindly according to local rules, ant colonies can perform collective tasks which are far beyond the capacities of their constituent components (Hölldobler – Wilson, 1990).
[18] The swarm program and the swarm software were launched in 1994 by Chris Langton at Sante Fe Institute in New Mexico. The *Swarm* approach differs from *Ants* because the basic architecture of the Swarm is the simulation of collections of *concurrent* agents.
See, for more: http://www.swarm.org/intro.html,
and: http://mitpress.mit.edu/journal-home.tcl?issn=10645462
[19] The Floys approach is similar to the Ants and Swarm approaches, but it considers flocking creatures characterized by collective flying or flocking and territorial instinct that act following simple local rules.

The more advanced applets allow changing traits and the personality of individual Floys (iFloys & eFloys), and also breeding and evolution in the population (eFloys).
For more details, see: http://www.aridolan.com/JavaFloys.html

[20] The *recursive* approach considers many phenomena observed in populations (growth and diffusion) that give rise to unexpected patterns as the result of a recursive application of simple local syntactical rules (alphabet and syntax), often defined in a qualitative way. Such an approach is often an application of the cellular automata one. As an example, we can refer to *L-systems* - short for "Lindenmayer System", after Lindenmayer [1972] - that model growth processes which arise from the application of sets of rules over symbols (also known as "formal grammars") (Prusinkiewicz – Lindenmayer, 1990; Green, 1993, Holland, 1998).
For more specifications see: http://www.cpsc.ucalgary.ca/projects/bmv/vmm/title.html

[21] We may also include in the class of recursive systems the *fuzzy systems*, deriving from the fuzzy sets theory introduced by Zadec (Zadec, 1965, Negoita, 1981; Cox, 1994).

[22] *"Genetic Algorithms (GAs) were invented by John Holland* (1975) *and developed by him and his students and colleagues. This led to Holland's book "Adaption in Natural and Artificial Systems" published in 1975.*
In 1992 John Koza [1992] *used genetic algorithms to evolve programs to perform certain tasks. He called his method "genetic programming" (GP). LISP programs were used, because programs in this language can be expressed in the form of a "parse tree", which is the object the GA works on.*
See: http://cs.felk.cvut.cz/~xobitko/ga/
For more details: http://www.cs.cmu.edu/Groups/AI/html/faqs/ai/genetic/top.html
http://pscs.physics.lsa.umich.edu/complexity.html,
http://home.online.no/~bergar/mazega.htm.

[23] The micro-macro feedback is often positive, in the sense that it amplifies the initial casual impulse. At other times it is negative and tends to order the behaviour of the system as a whole by pegging the micro behaviour to the macro behaviour; or, vice-versa, by eliminating the micro behaviours that are deviant with respect to the macro behaviour.

[24] In complex systems theory the feedback is considered between agents and not as a determining feature of the system.
See: http://pscs.physics.lsa.umich.edu/complexity.html,
http://home.online.no/~bergar/mazega.htm.

[25] See: di Primio (1999).
See also: http://ais.gmd.de/~diprimio/bar/workshops/ws4/plain/BAR-Poster-fdp.html

[26] The global "cooperation" of the agents of a dynamic system which spontaneously emerges when an attractor state is reached is understood as self-organization. Speaking of an attractor makes sense only in relation to its dynamic system; likewise, the attractor landscape defines its corresponding dynamic system (Albin, 1998).
For details, see: http://www.c3.lanl.gov/~rocha/ises.html.
See also: von Foerster, 1960; Haken, 1977; Prigogine, 1985; Kauffman, 1993.

[27] This is the case of populations of insects which act by creating an "aromatic potential field" by spreading *pheromones* or other permanent messages. With their micro behaviours the agents spread pheromone in one site; the increasing concentration of pheromone increases the probability that each agent will move in the direction of that site. The micro-macro feedback is quite evident.
This sequence requires a certain number of insects. Only above a *critical activation mass* of insects can the pheromone amplify and become effective, and lead to some accumulation effect. See: Deneubourg – Goss, 1989.

[28] With combinatory systems, as with any system, in order for micro behaviours to be produced, and thus in order for macro behaviour to occur, a supply of energy resources is usually necessary. Alongside the intial inputs we must also consider the energy inputs of the system, which nevertheless must be kept distinct from the *initial impulse*. We must thus keep in mind that, in order to provide a technical explanation of the action of such systems, and above all for the purpose of planning them, knowledge of the energy inputs can turn out to be indispensable.

[29] The characteristic of *ramification* appears in combinatory systems, typically of the diffusion variety, that have a temporal dynamics during which part of the *base* is transformed into another combinatory system that can subsequently expand, and whose agents have certain features in common with the other system and others that are different.

We can thus imagine that a new branch takes off from the original system and in time can be reabsorbed or independently maintained. In the latter case other ramifications can subsequently occur.

On the description of the branchings of the living, see for example Maturana-Varela, 1987; the authors refer to branching with the meaningful term *genetic drift*. Monod, 1971 considers the combinatory system of evolution and justifies the branchings in terms of random mutations in the genotype and of uniform replications of the new genotype; the resulting phenotype change is spread if it has advantages for life; otherwise it tends to disappear.

[30] With the *expansion* effect, the micro and macro *rules* that initially operated on a limited base of **N** agents are extended to an open set of agents.

[31] Lindgren, 1997

[32] On path dependence and Polya urns see Arthur, 1994.

[33] All the models may be algebraically rearranged and simplified.

[34] We can of course suppose that the probability of speaking is dependent on the rate of talking people, so that $p_R(n, t)$ can be written as $p_R(n, N, t)$.

We can, on the contrary, simplify the model by supposing time is continuous and all the talking people are equal in their micro behaviour, ignoring the probabilities $r_p(t)$, $s_p(n)$ and $b_R(n)$:

$$\textbf{[D]} \quad \begin{cases} \mathbf{v}(t_0) \leftarrow \text{“CHANCE”} \\ \mathbf{M}(t) = [\mathbf{k}\,\mathbf{v}(t) + \mathbf{Q}\, r\,]\,(1 - \mathbf{a}) \\ \mathbf{v}(t) = w\,\mathbf{M}(t) + \mathbf{v}_{min} \end{cases}$$

[35] We can conceive of progress as a consequence of evolution. On the difficulty of developing a theory of progress, see: Heylighen and Bernheim, McCarthy.

[36] The International Index of Social Progress (ISP) and the Weighted Index of Social Progress (WISP) are well-established tools developed since 1974 by Richard Estes of the University of Pennsylvania's social work faculty. See: http://newciv.org/GIB/BOV/BV-377.HTML

The index measures and aggregates 46 different factors for each nation.

For progress in health, go to:

http://www.chicagometropolis2020.org/indicators/cm-2020/community/commun1.htm

For progress in human and family conditions, see: http://www.cabq.gov/progress/goalist.html .

[37] These systems are examples of the mechanism of increasing return in collective phenomena (Arthur, 1994).

[38] The *classic explanation* states: «Phenomenon F has these characteristics because the conditions C exist and because laws L and theories T apply». The explanation is *valid* if the set of assumptions imply as a conclusion precisely the phenomenon being studied.