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Self Organization and Chaos in Collective Phenomena The Combinatory Systems View

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Abstract

The macro behaviours of *collectivities* – that is unorganized *social systems* (populations, collective units, social totalities, groups, pluralities, collections, matrices, etc.) - can produce many important phenomena four of which are: the accumulation of objects, the spread of features or information, the pursuit or exceeding of a limit, and the attainment and maintenance of an order among the micro behaviours. A very relevant fifth effect, which includes the others, is the interdependent dynamics of individual *improvement* and collective *progress* in the overall state of a collectivity (defined in opportune ways).

If we accept the traditional definition of *self-organization* as a macro behaviour in which the micro behaviours appear to be directed, or organized, by an *invisible hand* in order to produce the *emerging phenomenon* represented by the formation of ordered structures, of recognizable patterns, then all five of the collective phenomena can be defined as self-organization.

To understand, explain and, to a certain extent, control these collective phenomena I suggest the simple Theory of Combinatory Systems.

By *Combinatory System* I mean an unorganized system made up of a plurality of similar elements; the macro behaviour of the system, as a unit, derives from the *combination* of the analogous behaviours of its similar elements, according to a feedback relation between micro and macro behaviours.

This internal feedback between micro and macro behaviours guarantees the maintenance over time of the system's macro behaviour and directs the micro behaviour. When the system starts up "by chance", it then maintains its behaviour "by necessity", as if a Supreme Authority regulated its time path and produced the observable effects and patterns.

This paper presents the fundamental ideas and mechanisms that underlie these systems and some

models showing the action of the *self-organization*.

Because of the extreme wealth of literature on systems I've made the choice to mention only the fundamental and well-known (or the very specific) contributions and web sites.

Keywords: combinatory systems, micromotives and macrobehaviour, populations and collectivities, path dependence, chaos in economics

1. Introduction. Macro behaviour of collectivities. Two different approaches

I define *collectivity* (social system, population, collective unit, social totality, group, plurality, collection, matrix, and so on) as a set of similar, unorganized elements, or agents, that produce analogous micro behaviours (which lead to similar micro effects), but which, as a whole, produce a macro behaviour (and at times a macro effect or a recognizable pattern) which is not included in advance in the operating programme of the agents' behaviour.

Collectivities have always been a very complex subject of study, and for this reason a fascinating and interesting one as well.

If observed *from a certain distance* collectivities appear distinct with respect to the individuals, and thus seem able to produce an autonomous macro behaviour due to the interactions of the micro behaviours.

Since Schelling's [1978] attempt, in the very famous *Micromotives and macrobehaviour*, to offer a logical explanation for *collective macro behaviours* shown by intelligent agents, and Conway's discovery of the fantastic world of *Life*, the study and the simulation of the behaviour of collectivities has become a fruitful field of research ¹.

It is rather difficult to provide a list of theories, models and instruments usefully available to explore this research field; without pretending to be exhaustive we can classify the studies of collectivities into two main kinds of approaches:

1. the *macro* or *analytic approaches* which aim to build models of systems capable first of all of justifying the *macro behaviour*; the *micro behaviours* are considered unobservable or not important because the relations that link the elements are too complex and numerous; the abundance of connections make the construction of meaningful models based on elements very difficult; so the macro approach produces a macro description of the behaviour of collectivities. Included in this typology are:

- a. the *cybernetic* approach,

- b. the *systems dynamic* approach,
- c. the *complex systems* approach,
- d. the *adaptive complex systems* approach,
- e. the *synergetic* approach,
- f. the *autopoietic* approach;

2. the *micro* or *syntetic approaches* whose models are built exclusively (or prevalently) by studying the micro behaviours and the micro rules which connect them ². The macro behaviour is a consequence – often unexpected – of the action of these connections. Forming part of this typology are the models worked out by the *Cellular Automata Theory*, which allow us to explore the systems by simulating *Artificial Life*. We can consider, in particular:

- a. the *Cellular Automata* approach,
- b. the *Alife* approach;
- c. the *Ants* approach;
- d. the *Swarm* approach,
- e. the *Floys* approach,
- f. the *Genetic Algorithms* approach.

It is clear that the progress in the computer simulation of the behaviour and evolution mechanisms and the first technological realizations (artificial life environments, robots, intelligent toys, self-reproducing machines, agents on the web) are creating the basis for a new age: the coming of *artificial beings* and *artificial societies* ³.

2. The macro approaches. A short survey

We can consider in this approach:

1) The *cybernetic approach*, in particular *evolutionary cybernetics*, which aims to develop a theory to explain the process of arranging components to form a pattern different from what could occur *by chance*, by some criterion or better than it was before, and attempts to provide generalizations about how cells, or organisms interact or how and why people work together and form larger unities ⁴.

We can include in the *cybernetic approach* the models of *population dynamics*, which try to represent population dynamics in terms of the dynamics of the number of elements, using *Malthusian models* and *Volterra-Lokte* equations, in various forms ⁵.

2) The *systems dynamic* approach – which may be viewed as an extension of the *cybernetic* one – is a method and a technique for understanding how the behaviour of concrete collectivities arise and change over time. Internal feedback loops within the structure of the system influence the entire system behaviour ⁶.

3) The *complex systems approach* is a new field of science studying the *collective* behaviour of many basic but interacting units ⁷. “*To be more precise, our definition is that complexity is the study of the behaviour of macroscopic collections of such units that are endowed with the potential to evolve in time. Their interactions lead to coherent collective phenomena, so-called emergent properties that can be described only at higher levels than those of the individual units*” ⁸. There are three interrelated approaches to the modern study of complex systems: (1) how interactions give rise to patterns of behaviour; (2) understanding the ways of describing complex systems; and (3) the process of formation of complex systems through pattern formation and evolution ⁹.

4) The *adaptive complex systems approach*, which studies how complex systems interact and exchange information with their environment to maintain their internal processes over time ¹⁰ and to develop some form of cognition ¹¹. Here, we will use the term "complex adaptive system" to refer to a system with the following properties:

- a collection of primitive components, called "agents" ¹²,
- interactions among agents and between agents and their environment,
- agents adapt their behaviour to other agents and environmental constraints,
- as a consequence, system behaviour evolves over time,
- unanticipated global properties often result from the interactions ¹³.

The theory of adaptive complex systems presents three types of analysis (we continue, of course, to speak in general terms):

a. the analysis is aimed mainly at the external dynamics of the system; that is, the *macro behaviour*. The emerging properties are studied in terms of macro variables associated with the macro behaviour. This is the *synergetic* approach;

b. the analysis studies how adaptive complex systems develop and maintain over time the network of vital processes. This is the *autopoiesis* approach;

c. the analysis considers the component elements of adaptive complex systems as *species* and studies the processes of adaptation to environmental variations (a succession of micro mutations) that follow from the reciprocal local interactions according to rules established over time or variants based on other well-known rules. This is the *Artificial life*

(A-life) *approach* that employs genetic algorithms, cellular automata, neuronal networks, and intelligent agents (L-systems, ants, swarms, etc.).

5) In the area of *adaptive complex systems*, *Haken's synergetics* approach is the theory of *cooperative behaviour* in active systems¹⁴. The *synergetics approach* provides an exogenous description of complex systems without entering into internal operative mechanisms and without examining the micro and macro rules from which the behaviour originates¹⁵. In other words, in the *synergetics* approach *Complex systems* are composed of a number of elements that is so high they can only be analysed and described in terms of a limited number of parameters defined as order parameters.¹⁶

6) In the area of *adaptive complex systems*, the *autopoietic* approach is the theory of *self-maintaining* systems¹⁷. The *autopoietic* approach to *adaptive complex systems* is based on two fundamental concepts: 1) the idea of *behavioural coupling*; 2) the idea of *operational closure* of the system. Whereas in complex systems the behaviour of each agent may be thought of as depending on the behaviour of other agents and on the collectivity in general, we can argue that "*...the autopoietic conduct of an organism A becomes a source of deformation for an organism B, and the compensatory behaviour of organism B acts, in turn, as a source of deformation for organism A, whose compensatory behavior acts again as a source of deformation for B, and so on recursively until the coupling is interrupted.*"¹⁸. So, during the course of *behavioural coupling*, each participating autopoietic agent is, with respect to the other(s), a source (and a target) of compensatory perturbations for each other. These are 'compensatory' in the sense that (a) there is a range of 'compensation' bounded by the limit beyond which each system ceases to be a functional whole and (b) each iteration of the reciprocal interaction is affected by the one(s) before¹⁹.

3. The micro approaches. A short survey

The common fundamental idea of the *micro approaches* is that the *macro* behaviour of collective systems is determined by the interaction of the *micro* behaviour of the *agents* which form the system, acting isolated or in teams²⁰. The only reasonable approach to complexity in such systems is synthetic: to recognize or to define the *micro rules* which produce or direct the micro behaviours. In other words: *not to describe a complex system with*

complex equations, but let the complexity emerge by the interaction of simple individuals following simple rules.

The synthetic approach is based on three main ideas:

- the idea that complex, sophisticated, adaptive solutions can be generated by automatic, blind, knowledge-lacking mechanisms (Evolution).
- the idea that complex systems, such as life, are actually the emergent behaviours of systems with many elements that operate according to simple, local rules (Artificial Life).
- the idea that a personal computer can be an important scientific laboratory tool, and that new insights and new knowledge can (potentially) be achieved by using inexpensive equipment for conducting scientific experiments from one's home ²¹.

We can include in this approach:

1) The *cellular automata* approach, which may be considered the most general approach to simulate behaviours in collectivities; the theory of cellular automata builds mathematical models of a system which consists of an array of cells (possibly in more than one dimension) ²². A set of rules defines the transition from one state to another from one step in the time frame. It is important to note that the rules that define the micro behaviour of a cell are only *local rules*, in the sense that the state of the cell depends only on one of a specified number of neighbours and not on the state of the array ²³.

2) The *Alife* approach may be considered a specific case of the *cellular automata* approach, because it refers to cells simulating simple *living autonomous reactive agents* ²⁴ to show how interactions among neighbouring agents, following local rules, lead, at a higher level, to complex patterns by self-organization ²⁵.

The Alife approach presents many interesting variations, depending on the supposed nature of the agents. The most well-known are: Ants, Swarm and Floys.

3) The *Ants* approach considers ant colonies as collections of reactive agents in order to study the self-organization and sociogenesis in ant and in wasp colonies ²⁶. Although each ant is characterised by limited capabilities (limited local movement, recognizing food or ants, marking territory with chemical traces and so on) and acts blindly according to local rules, Ant colonies can perform collective tasks which are far beyond the capacities of their constituent components ²⁷.

4) The *Swarm* approach shows logic and patterns of behaviour similar to *Ants* but differs from them because the basic architecture of the Swarm is the simulation of collections of *concurrent agents* ²⁸.

5) The *Floys* approach is similar to the Ants and Swarm ones, but considers flocking creatures characterized by collective flying or flocking and territorial instinct that acts following simple local rules. They differ from most other flocking Alife animals by having the following properties:

- territorialism (they defend their territory against intruders)
- potential individualism (each can possess a different personality)
- ability to evolve (using a Genetic Algorithm code) ²⁹.

6) The *recursive* approach considers many phenomena observed in populations (growth and diffusion), giving rise to unexpected patterns as the result of a recursive application of simple local syntactical rules (alphabet and syntax), often defined in a qualitative way. Such an approach is often an application of the cellular automata one. As an example, we can refer to *L-systems* that model growth processes which arise from the application of sets of rules over symbols (also known as "formal grammars") ³⁰ and to fuzzy systems ³¹.

7) The *genetic algorithms* approach represents a model of *machine learning* which simulate its behaviour following the metaphor of the *processes of evolution* in nature ³². The machine works with a *population of individuals* represented by a set of character strings (or chromosomes). A recursive process of *crossover* operations – generally stochastic – simulate the reproductive behaviour; a defined environment generates the selection process as a function of the *fitness measure* of the individual that is supposed to compete with other individuals in their environment. Some *genetic algorithms* use a function of the *fitness measure* to select individuals (probabilistically) to undergo genetic operations such as crossover or *reproduction*, and this leads to the propagation of unaltered genetic material ³³.

4. Self-organization in collectivities. The Combinatory Systems approach

The macro behaviours of the collectivity can produce many important phenomena or effects, four of which are: the accumulation of objects, the spread of features or information, the pursuit or exceeding of a limit, and the attainment and maintenance of an order among the micro behaviours. A very relevant fifth effect, which includes the others, is the interdependent dynamics of individual *improvement* and collective *progress* in the overall state of a collectivity (defined in opportune ways).

If we accept the traditional definition of *self-organization* as a macro behaviour in which

the micro behaviours appear to be directed, or organized, by an *invisible hand* in order to produce the *emerging phenomenon* represented by the formation of *ordered structures*, of recognizable patterns, then all the collective phenomena mentioned before can be defined as *self-organization* ³⁴.

To understand, explain and, to a certain extent, control these collective phenomena I suggest the simple Theory of *Combinatory Systems*.

In plain words, by Combinatory System I mean an *unorganized system* made up of a plurality of *similar elements*; the macro behaviour of the system, as a unit, derives from the “combination” of the *analogous behaviours* of its similar elements, according to a *feedback* relation between micro and macro behaviours.

This *internal feedback* between micro and macro *behaviours* – or between their micro and macro *effects* – guarantees the maintenance over time of the system’s dynamics. When the system starts up “by chance” it then maintains its behaviour “by necessity”, as if a *Supreme Authority* regulated its time path and produced the observable effects and patterns.

The existence of the micro-macro feedback is the condition for a complex system to be conceived as a combinatory system.

5. A general model of combinatory system

In order to give a simple illustration we shall indicate by $\mathbf{S}(t, \mathbf{N}) = [t, A(1), \dots, A(n), \dots, A(\mathbf{N})]$ a non-ordered system formed by \mathbf{N} agents (or elements), $A(n, t)$, observed for $t \in T$, appropriately defined ³⁵.

Let us suppose that each $A(n, t)$, $1 \leq n \leq \mathbf{N}$, has a *state* – denoted by an opportune set of variables – and also that it can change its state for $t \in T$, showing a micro behaviour as the movement of the state values in T .

Thus we can write $\mathbf{mb}(n, t)_{t \in T}$ for the *micro* behaviour of the element $A(n)$ observed in period T .

Let us also suppose that we can define $\{\mathbf{C}_{1 \leq n \leq \mathbf{N}} [\mathbf{mb}(n, t_h)]\}$ for a *combination* of those micro behaviours, at time t_h , where $\mathbf{C}_{1 \leq n \leq \mathbf{N}}$ indicates a set of *combination operation(s)*, appropriately specified (sum, product, average, min, max, etc.), of values for state variables associated with the \mathbf{N} elements ³⁶.

Moreover, we write $\mathbf{MB}(t_h) = \mathbf{F} \{\mathbf{C}_{1 \leq n \leq \mathbf{N}} [\mathbf{mb}(n, t_h)]\}$ to represent the *macro* behaviour of

$S(t, \mathbf{N})$, defined as a *recombining function* \mathbf{F} (or *macro rule*) of the combination of the micro behaviours, and $\mathbf{mb}(n, t_{h+1}) = \mathbf{f}_n \{ \mathbf{N}_n[\mathbf{MB}(t_h)] \}$ to represent the micro behaviour, where \mathbf{N}_n represents the *necessitating operation(s)* that link(s) the micro behaviours to the macro behaviour (or the micro and macro effects).

The combinatory system, observed on a discrete time scale, can be represented as follows
37.

$$\{ \mathbf{mb}(n, t_0) \leftarrow \text{“CHANCE”} \quad 1 \leq n \leq N \quad [\text{A.1}]$$

$$[\text{A}] \{ \mathbf{MB}(t_h) = \mathbf{F} \{ \mathbf{C}_{1 \leq n \leq N} [\mathbf{mb}(n, t_h)] \} \quad h = 0, 1, 2, \dots \quad [\text{A.2}]$$

$$\{ \mathbf{mb}(n, t_{h+1}) = \mathbf{f}_n \{ \mathbf{N}_n[\mathbf{MB}(t_h)] \} \quad 1 \leq n \leq N \quad [\text{A.3}]$$

Equation [A.1] shows that the first input is considered to be the product of *chance*.

In equation [A.2] I have indicated the same time reference (t_h), since usually the macro behaviour is *contemporaneous* to the micro behaviours, as it is derived from these.

In model [A] we assume absolute independence of the *macro behaviour* from the past history of the system; in fact, equation [A.2] simply describes the macro behaviour as a *function* \mathbf{F} of the *combination* of the micro behaviours.

Equation [A.3] instead describes how the subsequent *micro behaviour* $\mathbf{mb}(n, t_{h+1})$ depends on the past macro behaviour (again referring to t_h), according to a *necessitating function* \mathbf{f}_n (or *micro rule*) that we assume is specified for every $A(n, t)$ and according to the *necessitating operation(s)* represented by \mathbf{N}_n .

If a probability is associated with the transition of state of each element, then the combinatory system is *stochastic*; the macro behaviour depends on the probabilistic micro behaviours. In the opposite case it is *deterministic*.

A schematic model of combinatory system is shown in figure 5.

6. Irreversible and reversible combinatory systems. Path dependence and chaos

The preceding models [A] are different depending on whether or not the system is deterministic or probabilistic.

In *social* combinatory systems, whose structure is composed of *cognitive agents*, that is of

elements that can receive information from other agents and can decide to change their state, the probabilities play an essential role for understanding and modelling the systems.

In *probabilistic combinatory systems* the micro behaviour depends on a *probability of transition of state*, and is carried out in a *period of transition* of state.

Both *probabilities* and *periods* of transition of state nevertheless depend on the state of the system, so that, in turn, the *micro behaviours* are conditioned by the *macro behaviour* of the entire system.

The *probability of transition* should offer numerical information on all the characteristics observable, or even imaginable, in the element $A(n, t)$, such as to make a change of state possible, plausible, probable, likely. This thus expresses the influx of *necessitating factors* that impose on $A(n, t)$ its own micro behaviour. In other words, it should express the likelihood of a given micro behaviour and a given micro effect which can potentially be carried out and obtained from $A(n, t)$.

Due to the existence of the micro-macro feedback, if the state of the system derives from the state of its elements, this nevertheless influences the micro behaviours and the states of the elements in the base according to the probability of transition for each one; a probability that depends, in turn, on the state of the system.

We must therefore take account of this feedback, for example by writing that:

1) the state of each element depends on the probability that characterizes it; but this probability is in turn a function of the state of the system;

2) the length of the period of transition of state of each element that is modified is also a function of the state of the system.

The combinatory systems that are most interesting and easiest to represent are the *irreversible* ones, where both the micro and macro behaviour produce permanent effects by a process of imitation and social learning (residential or industrial settlements, the maintenance of the language, the spread of epidemics)³⁸.

Irreversible systems explain almost all the cases of *path dependence*, as we can see from [A.1] and [B.1] in the previous models³⁹.

In regard to combinatory system theory, recognizing the phenomenon of *path dependence* is not a theory but simply the observation that the dynamic of a social system – its macro behaviour or its macro effect – can be thought to depend on initial chance (dependence from initial conditions) and on the *recombining* rules of the micro behaviours of the agents.

Thus, the individual choices of the agents lead to micro behaviours deriving from the past history, that is from the macro behaviour (history dependence).

In this sense the *path dependence* is the proof of the action of the micro-macro feedback, even if path dependence theory does not include this mechanism in the explanation of the path dependence.

The deterministic action of path dependence, the necessity, is not a consequence of the past evolution of the path of the system, but of the micro-macro feedback, and then of the necessitating and recombining factors.

Ignoring micro macro feedback leads to a second consequence: the *path dependence* theory focuses particularly on the micro behaviour, considering the macro behaviour as a constraint to the individual freedom to decide.

The Combinatory Systems Theory also considers *reversible systems*; that is, systems whose elements may again show a state that occurred in the past, so that they may present a *cyclical* behaviour and, under certain conditions concerning the probability function regarding the transition of state of the elements, a *chaotic* one as well ⁴⁰.

Examples of *reversible* systems are those of diffusion and dissemination (fashion and contagion), whose elements may present a state chosen in a "repertoire" ⁴¹.

7. An example - The chaotic behaviour in reversible probabilistic systems

As an example, consider the case of a non-ordered system where every $A(n, t)$ is a Bernoulli random variable that, at any $t \in T$, shows only two states: $\mathbf{mb}(n, t) = [1 \text{ or } 0]$.

The macro behaviour is $\mathbf{MB}(t) = N(t)$, $0 \leq N(t) \leq N$, since for [A.2] we have simply established that $\mathbf{R}_{1 \leq n \leq N}[\mathbf{mb}(n, t)] = \sum_{1 \leq n \leq N} [\mathbf{mb}(n, t)]$.

We also suppose that the *probabilities* of transition from state "0" to state "1", $p(n, N)$, are defined for each $A(n, t)$ and for each $0 \leq N(t) \leq N$, as well as the probability $q(n, N) = 1 - p(n, N)$; to simplify, these probabilities might be assumed to be the same for each element, so that we write $p(N) = 1 - q(N)$.

We assume there is a *feedback between the micro and macro behaviour*, in the sense that the state of each element depends on the probability $p(n, N)$, which in turn depends on the state of the system, $N(t)$, which defines the macro behaviour.

Let us simply assume that the function $p(n, N)$ takes on the following values:

$$p(n, N) = p(N) = 2(N/N) \text{ if } 0 < N \leq N/2$$

$$p(n, N) = p(N) = 1 - [(2N - N)/N] \text{ if } N/2 < N \leq N.$$

If we simulate the micro behaviour by some experiment that generates *random numbers* for each element, we observe that after the random initial impulse that shapes $mb(n, t_0)$, the combinatory system presents a chaotic macro behaviour $\mathbf{MB}(t) = N(t)$.

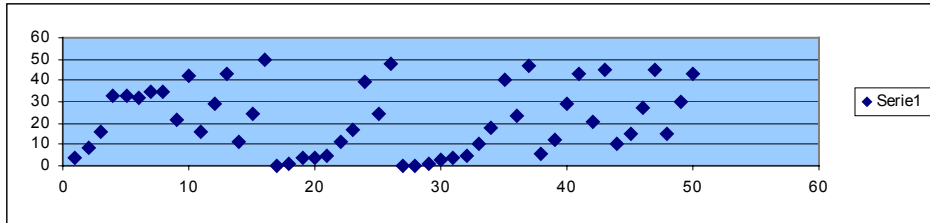
Figure 1 shows the results of three simulations generating the macro behaviour of the system outlined above, supposing $N = 50$ and $N(1) = 4$ due to initial *chance*.

Figure 2 shows the simulations of the macro behaviour changing the initial state.

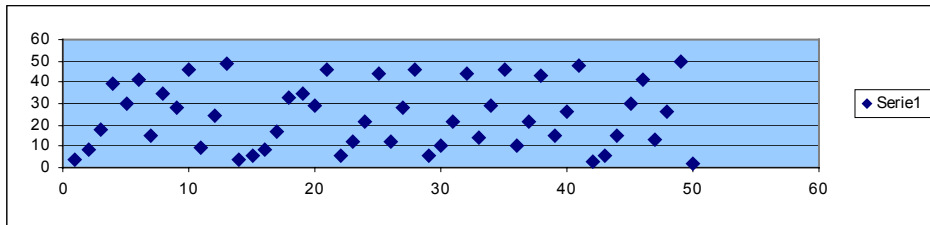
Figure 3 represents the results of the simulation of the system assuming that the function $p(n, N)$ increases straight line and assumes the value 1 for $N(t) = (4/5 N)$ and then decreases straight line to 0.

Fig. 1 – Reversible probabilistic combinatory system with chaotic macro behaviour changing random numbers - $N = 50$ and $N(1) = 4$

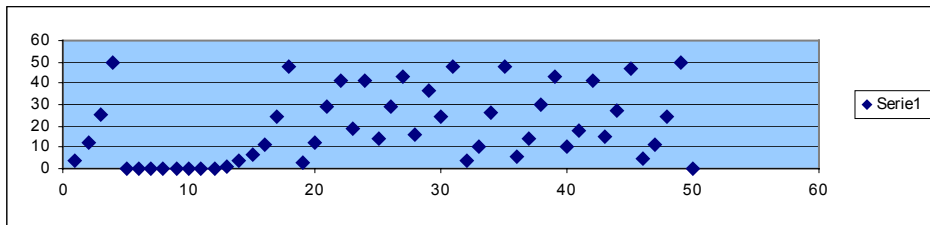
Test 1 - $N(0) = N(1) = 4$



Test 2 - $N(0) = N(1) = 4$ (new random numbers)



Test 3 - $N(0) = N(1) = 4$ (new random numbers)



Test 4 - $N(0) = N(1) = 4$ (new random numbers)

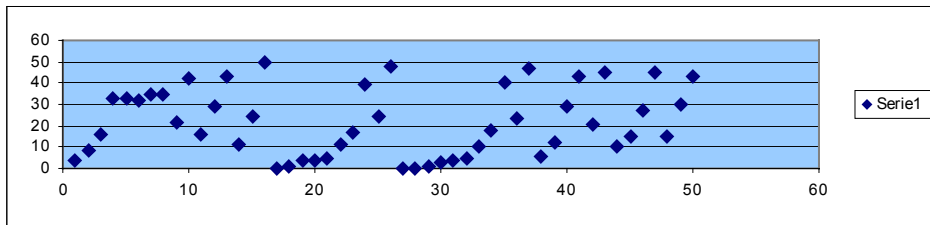
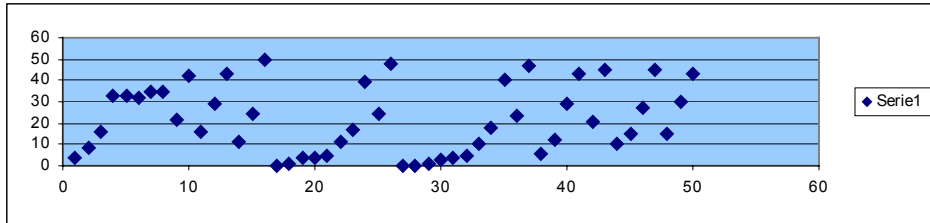
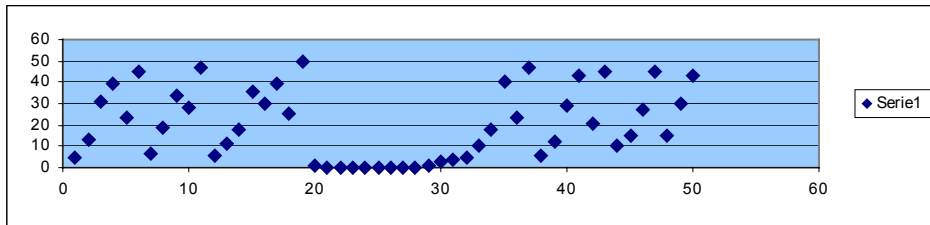


Fig. 2 – Reversible probabilistic combinatorial system with chaotic macro behaviour changing initial conditions, keeping the same random numbers - $N = 50$ and $N(1) = 4$

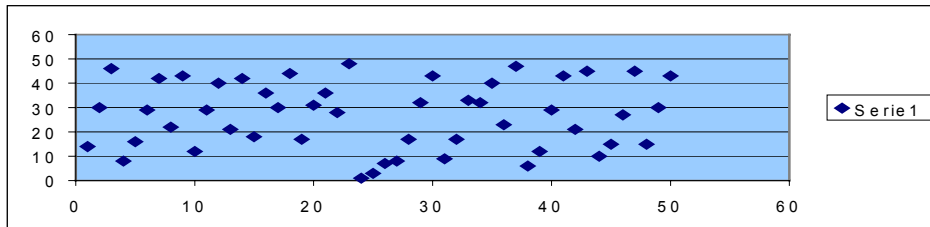
Test 1 - $N(0) = N(1) = 4$



Test 2 - $N(0) = N(1) = 5$



Test 3 - $N(0) = N(1) = 14$



Test 4- $N(0) = N(1) = 40$

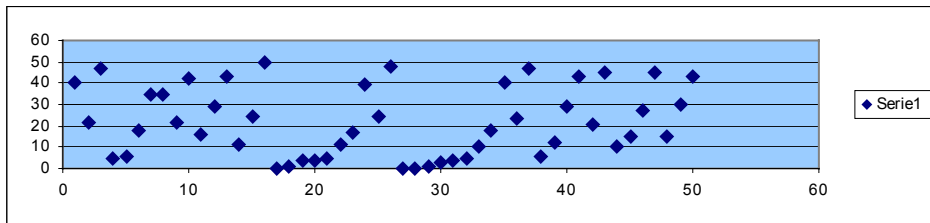
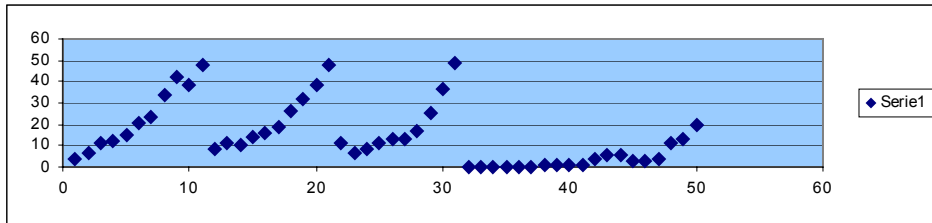
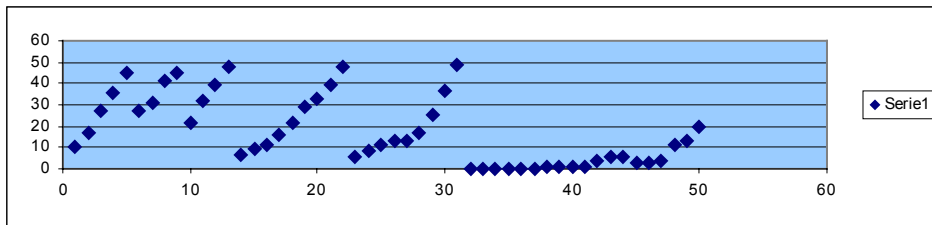


Fig. 3 – Reversible probabilistic combinatory system with chaotic macro behaviour changing probabilities - $N = 50$ and $N(1) = 4$. Probability increases straight line to 1 for $N = 40$ and then decreases to 0 for $N=50$.

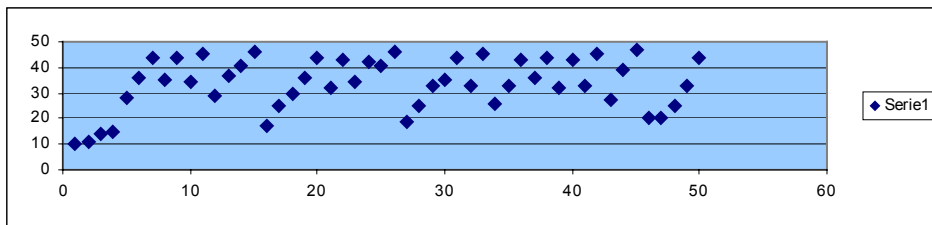
Test 1 - $N(0) = N(1) = 4$



Test 2 - $N(0) = N(1) = 10$ (same random numbers of the previous test)



Test 3 - $N(0) = N(1) = 10$ (new random numbers)



Test 4 - $N(0) = N(1) = 45$ (same random numbers)

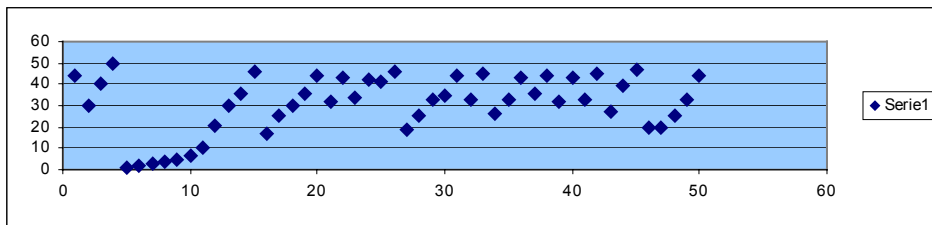
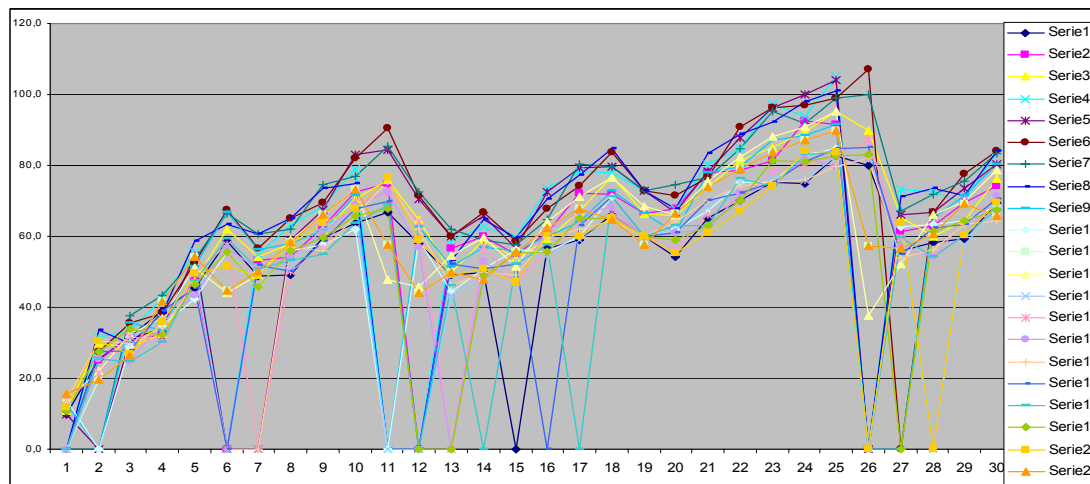


Figure 4 show the micro and the macro behaviours of the real reversible probabilistic combinatory system that produces the phenomenon of a murmur arising in a crowded room, which all of us can observe very easily.

Fig. 4 - Model of Murmur and Noise system with 20 agents



8. Conclusions. The main characteristics of the combinatory system approach

From the previous examples it emerges that combinatory systems may be viewed as *recursive complex systems* which differ from other models that explore collectivities – models that we have summarized in the macro and micro approaches (evolutionary cybernetics, autopoiesis, cellular automata, ants, swarm, etc.) – with regard to some fundamental characteristics.

The *first characteristic* concerns the internal structure of combinatory systems: they are *non-organized systems* that show a *self-organization*.

Systems, in general – and behavioural ones in particular – can be observed from both an exogenous and endogenous point of view.

Observing the system from an *exogenous* point of view means that the observer is not interested in the nature of the elements and the relationships that compose the structure of the observed system. The exogenous perspective brings out solely the possibility of describing the dynamics of the system in terms of variables, whether these are associated with the

system as a unit characterized by inputs, outputs and external feedback connections (functional description) or associated with the states of the structure (structural description)⁴².

When instead the observer analyses the nature of the elements that make up the structure of the system, he places himself in an endogenous perspective⁴³; that is, he undertakes an approach that leads him to "enter into" the system in order to search for and analyze the different nature of the elements that form its internal structure - and characterize its state - allowing them to interact and produce the micro-macro feedback and produce the structural behaviours.

From this point of view, and with regard to the nature of the elements in the structure, it is possible to distinguish among *organized*, or operative systems, and *non-organized*, or combinatory systems.

We define as organized those behavioural systems whose structure is composed of elements, defined as *organs*, which differ from each other, since they have a precise spatial and temporal *placement*, carry out a specialized *function* in relation to the entire structure, and have a specific *functionality* that delimits the admissible interactions with the other organs.

With endogenous observation, on the other hand, *combinatory* systems are quite different from organized systems; they are made up of elements, defined as *agents*, which present a similar nature, or similar significance, and develop analogous interactions (behaviour, processes); these interactions, *combining* together, produce *emerging* effects (the macro behaviour or macro effects of this behaviour) with reference to the unit; since the elements are similar they do not constitute organs, and thus such systems are unorganized.

For this reason combinatory systems are clearly distinct from organized systems, even if they may show some form of *self-organization*, in the sense that the agents may adjust and specialize their micro behaviours and produce a macro behaviour that can lead to some macro phenomenon, macro effect, or recognizable pattern⁴⁴.

The second *characteristic* regards simply the mechanism by which combinatory systems develop the *self-organization* that produces the collective phenomena.

The *central idea* is that in combinatory systems the collective phenomena are produced from a *combination* of the micro behaviours of agents but, at the same time, they condition these as the result of a *micro-macro feedback* relationship that seems to guide the individual behaviours and produces the collective phenomena.

This approach is quite different from that of complex systems – and in particular from

the micro approaches – precisely in that the *operating rules*, describing the behaviour of the system, must in some way include not only *local rules* but also the *feedback* between the micro and macro behaviours⁴⁵.

This means that we can view a collectivity as a *combinatory system* only if the behaviour of agents is not exclusively determined by *local rules* but also by a general *micro-macro feedback* rule, so that we *must observe*, or *assume*, mutual interdependence: the micro behaviours produce the macro behaviour, but this influences the micro behaviours in a micro-macro feedback which acts for many cycles⁴⁶.

The *macro behaviour* – or its macro effects – may be thought of as a *dynamic attractor* to which the micro behaviour tend and modify over time⁴⁷. For this reason we cannot consider in general the ants, the swarm and, more generally, the cellular automata approaches as examples of combinatory systems, except in the case in which the macro effect may affect the micro behaviours of the agents in some way⁴⁸.

The combinatory systems approach is neither a macro approach nor a micro approach; it is a micro-macro approach.

Recognizing the existence of a micro-macro feedback is indispensable for interpreting collective phenomena as deriving from a combinatory system: the state of the system at a given time must depend on the state of its elements; but this in turn must depend on the state of the system. The micro-macro feedback generates a synergetic effect that produces self-organization and emerging macro behaviours, which are only attributable to the collectivity.

We can thus say that in combinatory systems the micro behaviours create a *pattern in the collectivity* – normally invisible to the agents (according to third characteristics) – and this pattern influences or determines the micro behaviour of the agents⁴⁹.

We must nevertheless recognize also that each agent is normally blind to the macro behaviour of the system while being aware of the micro behaviours of some other agents; from this we immediately see the *third characteristic* of the combinatory systems: they are *incomplete* and *limited* information systems:

– they are *incomplete information systems* in that each of the $A(n) \in S(t, N)$ produce their own micro behaviours without considering the macro behaviour of the unitary system as information (except as an extreme case of a completely observable macro effect);

– they are *limited information systems* in that the micro behaviour of $A(n)$ depends on information about the micro behaviours (which occur or is only expected or foreseen) of a limited number of other *neighbourhoods* of $A(n)$ (defined in an opportune way⁵⁰), exactly as

in a cellular automaton.

The third characteristic is not in contrast with the second; they simply derive from different points of view:

– from an *external* point of view, the observer must recognize the macro behaviour and the micro-macro feedback action in order to define and build a model of the combinatory system;

– from an *internal* point of view the agents normally are unaware of the macro behaviour and act according to limited information.

In many cases, however, this third characteristic seems to fail because we can observe agents acting according to some general pattern related to the system. There is no contradiction: we must simply distinguish between the micro and macro *behaviours* and micro and macro *effects* in the environment ⁵¹.

In these cases the micro behaviours of the agents are related to some observable macro effects and the micro and macro feedback operates between the *micro* behaviour and the *macro* effects.

The *fourth characteristic* is connected to the fact that the combinatory system – even if its behaviour is deterministic - usually requires a *random input* to begin the micro-macro feedback. The output is entirely determined by the structural dynamics of the system ⁵², according to the micro rules and the micro-macro feedback ⁵³.

The combinatory systems are closed systems; their dynamic is only due to the joint action of "chance" and "necessity"; they can thus also be called "*chance-necessity*" ⁵⁴ systems and as such are opposed to open systems, which are typically "cause-effect" systems ⁵⁵.

According to the fourth characteristic we may observe that, in general, combinatory systems begin to operate when there is a change in the state of a minimum number of elements, and it ceases to develop its macro behaviour when there is a change in the state of a maximum number of elements.

This means that the micro-macro feedback is manifested only if the number of the $A(n, t)$ that develop the micro behaviour exceeds a *minimum* number (*critical activation mass*) and remains below a *maximum* number (*critical saturation mass*), which is defined each time for each specific system.

We must now consider a *very important fifth characteristic*: combinatory systems generally are set off *by chance*, but if they reach the *critical activation mass* they maintain their dynamics *by necessity*, due to the presence of *necessitating* and *recombining factors*.

Therefore, *to interpret the activity of combinatory systems we need always to understand the nature of both the recombining factors and the necessitating ones* since, without the joint action of these factors, there would be no micro-macro feedback and the collective phenomena the theory tries to explain would not be produced.

Chance by itself is never enough to maintain the macro behaviour, only to set it off. A *necessitating factor* (a constraint, a rule, a condition, a law, a conviction, an imitative act, etc.) must operate on the single elements to force each of these to adapt its micro behaviour to the system's macro behaviour.

Often these *necessitating factors* result from obligation, imitation, convenience, utility, desire, or the *operative programme* of the individual elements. The agents can be aware of these (I want to adjust my step to the marching step of my companions) or not (I don't want to transmit the flu virus, but this takes place without my being aware of it)⁵⁶.

The existence of one or more necessitating factors is indispensable but not yet sufficient; the system must also be able to recombine the micro behaviours (or the micro effects) in order to produce the macro behaviour (or the macro effect); some *recombining factors* (rule, convention, algorithm, etc.) must operate in the system so that, through the micro-macro feedback, the necessitating factor can also operate.

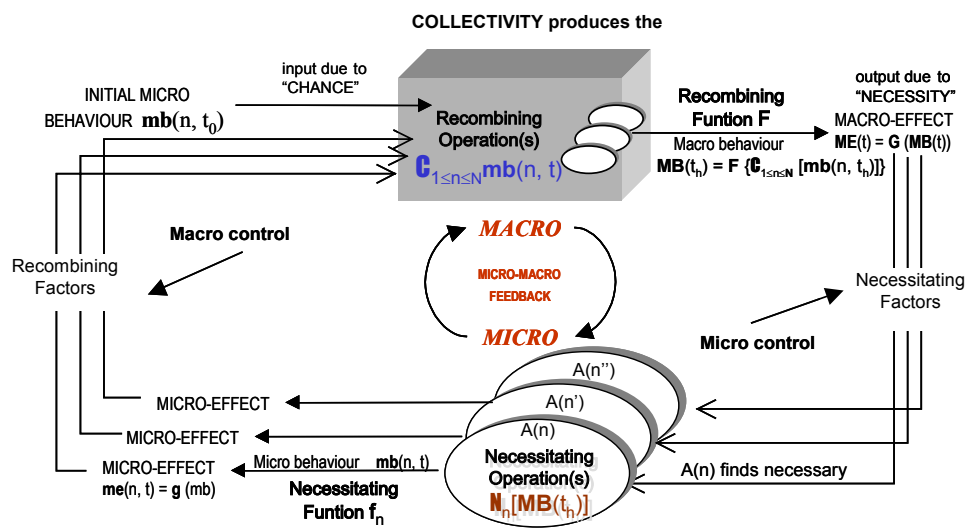
While the *recombining* factor characterizes the macro rules, the *necessitating* factor characterizes the micro ones.

In order to explain the activity of combinatory systems we must understand the nature of the macro rules, which specify the recombining factor, and of the macro rules, which specify the necessitating factor; the joint action of these factors gives rise to and maintains the macro and micro behaviours.

Other relevant characteristics (I mention only) concern the fact that, even though combinatory systems are unorganized and closed systems, they can organize⁵⁷ themselves into specialized subsystems and show ramifications⁵⁸ and can expand⁵⁹ their effects on elements belonging to a vaster environment.

Figure 5 presents a simple model that includes all the above-mentioned elements according to model [A].

Fig. 5 – The fundamental elements of a combinatory system



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Combinatory Sistem Theory: <http://www.ea2000.it/cst>

¹ The Game of Life was invented by John Horton Conway and described by Gardner [1970]. For more see: <http://serendip.brynmawr.edu/complexity/life.html - conway>.

The Life's game is an example of *Cellular Automata Theory*; it consists of a table of cells and a set of local and irreversible rules which describe how to find successive generations of the cells; the *game of Life* is thus an efficient tool to simulate the evolution of, for example, a colony of living organisms.

In the web we can find many applets (Java) to easily play the game; for instance:

<http://www.multimania.com/ldavid/indexe.html>

<http://bloch.ciens.ucv.ve/~felix/Java/Simulation/Conway/>

<http://hensel.lifepatterns.net/>

² The simulation of social behaviour by local rules may be defined as a “*Synchronic analysis [that] rules out the possibility of testing an important class of explanations, those based on process. If you want to understand why a person acts as she does, it is certainly possible to look around in the immediate environment for an explanation. But often an explanation needs to look also, or perhaps primarily, at events that occurred in the past and at how the present situation developed from previous circumstances.*”; see: Gilbert [1995]; see also: and Gilbert [1994]

³ For a general survey, see: <http://www.generativeart.com/>

⁴ The *evolutionary cybernetics* develops a complete philosophy or "world-view", based on fundamental Darwinian principles: "*Blind Variation and Selective Retention*" (BVSR) is a phrase introduced by D. T. Campbell [1960], as a way of describing the most fundamental principle underlying Darwinian evolution. (Campbell only applied it to the evolution of knowledge, but we here apply it in the most general context). The BVSR formula can be understood as a summary of three independent principles: blind variation, asymmetric transitions, and selective retention.”. See PRINCIPIA CYBERNETICA WEB, <http://pespmc1.vub.ac.be/BVSR.html>

⁵ Volterra [1926] suggested a simple model of a prey-predator type with the intent of explaining oscillatory phenomena relating to the co-evolution of some kind of fish in the Adriatic sea.

If **N(t)** indicates the population of the prey and **P(t)** the one of predator at time **t**, then the model may be written as:

$$\begin{array}{ll} \text{(prey)} & \mathbf{dN/dt = N (a - bP)} \\ \text{(predators)} & \mathbf{dP/dt = P (cN - d)} \end{array}$$

⁶ The field was developed initially from the work of Forrester [1961] as Systems (Industrial) Dynamics. Systems Dynamics is connected to Systems thinking, which looks at exactly the same kind of systems from the same perspective. It constructs the same causal loop diagrams. But it rarely takes the additional steps of constructing and testing a computer simulation model, and testing alternative policies in the model.

For more, see:

<http://www.albany.edu/cpr/sds/>

<http://sysdyn.mit.edu/home.html>

<http://www.uni-klu.ac.at/users/gossimit/links/bookmksd.htm>

⁷ Gell-Mann [1995] traces the meaning to the root of the word *complexity*. *Plexus* means braided or entwined, from which is derived *complexus*, meaning braided together, and the English word "complex" is derived from the Latin. Complexity is therefore associated with the intricate inter-twining or inter-connectivity of elements within a system and between a system and its environment.

“*There is no single Theory of Complexity, but several theories arising from the various sciences of complexity, such as biology, chemistry, computer simulation, evolution, mathematics and physics. The work referred to will be that undertaken over the past three decades by scientists associated with the Santa Fe Institute in New Mexico, and particularly that of Stuart Kauffman and John Holland on complex adaptive systems (CAS), as well as the work of scientists based in Europe, such as Prigogine, Sengers, Nicolis, Allen and Goodwin.*”. See E. Mitleton-Kelly [1997].

⁸ Coveney - Highfield [1995]

“*The underlying rules of the system are changing over time, which means that different agents behave according to different rules at different times. . . . Because of these difficulties, a class of models, variously called "artificial worlds", "particle-based", and "agent-based", have been a popular approach to studying CAS.*”

For more details: Forrest – Jones [1994]

⁹ A pattern is a property of the system as a whole but is not a property of small parts of the system. ...a property of a system that allows its description to be shortened as compared to a list of the descriptions of its parts. See, for example: <http://www.necsi.org>. See, also: Stacey [1995].

¹⁰ The term *Complex adaptive systems* is used by the Santa Fe scientists to describe complex systems which adapt through a process of self-organisation and selection. However, physical, chemical and biological systems are not conscious and do not 'learn' in the sense that humans learn. The term complex evolving system may be used to distinguish human from other complex systems. In particular, *Complex evolving systems* refers to those systems which are able to learn and which change their internal structure and organisation over time, thus changing the behaviour of individual elements. See: Allen [1997].

¹¹ "A cognitive system is a system whose organization defines a domain of interactions in which it can act with relevance to the maintenance of itself..."; see: Maturana - Varela [1980], p. 13.

¹² Agent may be defined as "A natural or artificial entity with sufficient behavioural plasticity to persist in its medium by responding to recurrent perturbation within that medium so as to maintain its organisation.". For a complete classification of agents, see: Goldspink [2000].

¹³ "Many natural systems (e.g., brains, immune systems, ecologies, societies) and, increasingly, many artificial systems (parallel and distributed computing systems, artificial intelligence systems, artificial neural networks, evolutionary programs) are characterized by apparently complex behaviours that emerge as a result of often nonlinear spatial-temporal interactions among a large number of component systems at different levels of organization."

See: <http://www.cs.iastate.edu/~honavar/alife.isu.html>

¹⁴ "The word *synergetics* is composed of Greek words meaning «working together».", Haken [1982], p. 2. I have specified that I am speaking of Haken's *synergetics*, to avoid confusion with the language used by R. Buckminster Fuller and E. Applewhite to construct a metaphysical description of the world.

The integration of geometry and philosophy in a single conceptual system providing a common language and accounting for both the physical and metaphysical.

What distinguishes Fuller's *synergetics* from more traditionally metaphysical discussions of the temporal versus the eternal, and from most contemporary philosophy, is Fuller's consistent use of geometric concepts to express such ancient dichotomies.

See, for more, Kirby Umer, *An Introduction to Synergetics*, in:

<http://www.teleport.com/~pdx4d/synhome.html>

¹⁵ "Synergetics is an interdisciplinary field of research. It deals with open systems that are composed of many individual parts that interact with each other and that can form spatial, temporal, or functional structures by self-organization. The research goal of synergetics is three-fold: (1) Are there general principles of self-organization? (2) Are there analogies in the behavior of self-organizing systems? (3) Can new devices be constructed because of the results in (1) and (2)? From a mathematical point of view, synergetics deals with nonlinear partial stochastic differential equations and studies their solutions close to those points where the solutions change their behavior qualitatively." Haken [1997].

Synergetics is defined as the science of co-operation, and Haken pioneered the scientific analysis of hierarchically organized co-operative phenomena in physics, with applications also in biology and the social sciences. He was also one of the early workers in chaos theory and self-organization and was one of the first to recognize co-operative self-ordering in various kinds of dynamic systems.

¹⁶ See, for example, Serra - Zanarini [1990], Corning [1995].

¹⁷ «A unit realized through a closed organization of production processes such that (a) the same organization of processes is generated through the interaction of their own products (components), and (b) a topological boundary emerges as a result of the same constitutive processes.» Zeleny [1981], p. 6. "A machine / system which is a member of the class of autonomous systems and which meets the requirement of being organized (defined as a unity) as a network of processes of production, transformation and destruction of components that produces the components which: (i) through their interactions and transformations regenerate and realize the network of processes (relations) that produced them; and (ii) constitute it as a concrete unity in the space in which they exist by specifying the topological domain of its realization as such a network. "

See: ENCYCLOPAEDIA AUTOPOIETICA WEB; <http://www.enolagaia.com/EA.html#>.

See: Maturana - Varela [1980]; Varela [1979].

¹⁸ Maturana - Varela [1980], p. 120. The idea of behavioral coupling is related, or derived from that of structural coupling. "In general, when two or more plastic dynamic systems interact recursively under conditions in which their identities are maintained, the process of structural coupling takes place as a process of reciprocal selection of congruent paths of structural changes in the interacting systems which result in the continuous selection in them of congruent dynamics of state.", Maturana - Guilloff [1980]. p. 139. "Phrased more succinctly, structurally-coupled systems ... will have an interlocked history of structural transformations, selecting each other's trajectories." See: Varela [1979]. pp. 48-49.

¹⁹ For more, see: <http://www.enolagaia.com/EA.html#S>

²⁰ The style of cooperation or competition between agents depends on the problem to be solved. The formation

of teams of agents is the expected approach to the organized division of labor, task-sharing or other methods of collective problem solving. See, for further details:

<http://borneo.gmd.de/AS/art/index.html>

²¹ For more details: <http://www.aridolan.com/>. With synthesis, the model-creator aims to accurately describe a system's components and plausible interactions, and then use a realization of that description as an empirical basis for the study of the system's global dynamics. The emphasis is put on finding appropriate abstractions for describing components and interactions rather than on finding abstractions that are useful for reasoning about global dynamics.

This *bottom-up* approach is called *Agent-Based Modeling* (ABM).

See, for more details: <http://www.swarm.org/csss-tutorial/frames.html>

²² "A cellular automaton can be thought of as a stylised universe. Space is represented by a uniform grid, with each cell containing a few bits of data; time advances in discrete steps and the laws of the "universe" are expressed in, say, a small look-up table, through which at each step each cell computes its new state from that of its nearby neighbours. Thus, the system's laws are local and uniform." For details, see: <http://www.brunel.ac.uk/depts/AI/alife/al-ca.htm>

²³ From the theoretical point of view, *Cellular Automata* (CA) were introduced in the late 1940's by von Neumann – A. W. Burks [1966].

From the more practical point of view it was more or less in the late 1960's when John Horton Conway developed the *Game of Life*. See, for more: Gardner [1970]; Dewdney [1989]; Dewdney [1990] (<http://www.csd.uwo.ca/faculty/akd/PERSONAL/hp.html>). Toffoli - Margolus [1987] and Ulam [1986], [1991]. The basic element of a CA is the *cell*. A cell is a kind of a memory element and stores – to put it simply - *states*. In the simplest case, each cell can have the binary states *1* or *0*. In more complex simulation the cells can have more different states. (It is even thinkable, that each cell has more than one property or attribute, and each of these properties or attributes can have two or more states.)

These cells are arranged in a spatial web - a *lattice*. The simplest one is the *one-dimensional* "lattice", meaning that all cells are arranged in a line like a string of pearls. The most common CA's are built in one or two dimensions.

Up to now, these cells arranged in a lattice represent a static state. To introduce *dynamics* into the system it is necessary to add *rules* to define the state of the cells for *the next time step*. In cellular automata a rule defines the state of a cell as a function of the *neighbourhood* of the cell.

For more details Schatten [1999].

²⁴ A reactive agent is a type of autonomous agent which:

- can send and receive messages;
- does not (in general) learn or "think";
- simply reacts to messages it receives in a stereotyped way (blind behaviour).

Agents may have:

- internal data representations (*memory* or *state*)
- means for modifying their internal data representations (*perceptions*)
- means for modifying their environment (*behaviors*).

²⁵ "The term "Artificial Life" is used to describe research into man-made systems that possess some of the essential properties of life. Artificial Life is often described as attempting to understand high-level behavior from low-level rules; for example, how the simple rules of Darwinian evolution lead to high-level structure, or the way in which the simple interactions between ants and their environment lead to complex trail-following behaviour. Understanding this relationship in particular systems promises to provide novel solutions to complex real-world problems, such as disease prevention, stock-market prediction, and data-mining on the Internet."

See, for more details: <http://alife.org/index.php?page=alife&context=alife> and <http://alife.santafe.edu/>

²⁶ Sociogenesis is meant as the process by which a single individual, a gravid queen, gives rise to an entire society of insects.

²⁷ "Ants occupy a central place in artificial life due to their relative individual simplicity combined with their relatively complex group behaviour.. They do so without being wired together in any specific architectural pattern, without central control, and in the presence of strong intrinsic noise. Ants can create architectural structures dynamically when and where they are needed, such as trails between nest and food sources, or "living bridges" when swarms of ants migrate in the rain-forest." Hölldobler – Wilson [1990].

²⁸ The swarm program and the swarm software were launched in 1994 by Chris Langton at Sante Fe Institute in New Mexico.

See, for more: <http://www.swarm.org/intro.html>,

and: <http://mitpress.mit.edu/journal-home.tcl?issn=10645462>

²⁹ “Floys belong to the flocking Alife creatures variety, sharing with them the social tendency to stick together, and the lifelike emergent behaviour which is based on a few simple, local rules. They differ from most other Alife flocking (Boids-type) implementations by being territorial animals that defend their territory against intruders. They are implemented as Java applets. The more advanced applets allow changing traits and the personality of individual Floys (iFloys & eFloys), and also breeding and evolution in the population (eFloys). “. For more details, see: <http://www.aridolan.com/JavaFloys.html>

³⁰ The name "L-system" is short for "Lindenmayer System", after Lindenmayer [1972], who was one of the first people to use syntactic methods to model growth. A simple L-system contains four elements:

1. **VARIABLES** are symbols denoting elements that can be replaced.

2. **CONSTANTS** are symbols denoting elements that remain fixed.

e.g. The expression

<subject> <verb> <predicate>

consists of grammatical variables. Each variable may be replaced by constants (English words or phrases) to produce sentences in English, such as "The cat sat on the mat" or "The dog ate the bone".

3. **RULES** ("syntax") define how the variables are to be replaced by constants or other variables. e.g. in the above example

<subject> - -> the cat

would be one such rule.

4. **START** words are expressions defining how the system begins. E.g. the above examples from English might start from the single variable

<sentence>.

For more specifications, see Prusinkiewicz – Lindenmayer [1990], Green [1993] and the site: <http://www.cpsc.ucalgary.ca/projects/bmv/vmm/title.html>

³¹ We may also include in the class of recursive systems the *fuzzy systems*, deriving from the fuzzy sets theory introduced by Zadeh (1965). The notion central to fuzzy systems is that truth values (in fuzzy logic) or membership values (in fuzzy sets) are indicated by a value in the range [0.0, 1.0], with 0.0 representing absolute Falseness and 1.0 representing absolute Truth. The fuzzy logic is summarized in these definitions:

– **Definition 1:** Let X be some set of objects, with elements noted as x . Thus, $X = \{x\}$.

– **Definition 2:** A fuzzy set A in X is characterized by a membership function $m_A(x)$ which maps each point in X onto the real interval [0.0, 1.0]. As $m_A(x)$ approaches 1.0, the "grade of membership" of x in A increases.

– **Definition 3:** A is EMPTY if for all x , $m_A(x) = 0.0$.

– **Definition 4:** $A = B$ if for all x : $m_A(x) = m_B(x)$ [or, $m_A = m_B$].

– **Definition 5:** $m_{A'} = 1 - m_A$.

– **Definition 6:** A is CONTAINED in B if $m_A \leq m_B$.

– **Definition 7:** $C = A$ UNION B , where: $m_C(x) = \text{MAX}(m_A(x), m_B(x))$.

– **Definition 8:** $C = A$ INTERSECTION B where: $m_C(x) = \text{MIN}(m_A(x), m_B(x))$.

See: Negoita [1981], Cox [1994].

³² “Genetic Algorithms (GAs) were invented by John Holland [1975] and developed by him and his students and colleagues. This led to Holland's book "Adaption in Natural and Artificial Systems" published in 1975.

In 1992 John Koza [1992] used genetic algorithms to evolve programs to perform certain tasks. He called his method "genetic programming" (GP). LISP programs were used, because programs in this language can be expressed in the form of a "parse tree", which is the object the GA works on. “. See:

<http://cs.felk.cvut.cz/~xobitko/ga/>

³³ For more details: <http://www.cs.cmu.edu/Groups/AI/html/faqs/ai/genetic/top.html>

John Holland pioneered the application of the process of natural selection to the problem of machine-learning in the form of the genetic algorithm (GA), which breeds possible solutions to problems founded on the Darwinian theory of natural selection. Based on *fitness* - that is, how well they solve a given problem - solutions from a population are bred together to produce new solutions. Solutions that perform badly die off, and those that perform well are bred again to produce even better solutions. “Genetic Algorithms (GA) are based on an evolution of random tries by 'individuals', not on logic as regular algorithms. It is a computer simulation of Darwin's theories. Though the whole process is built on randomness, the effect is not. It moves towards the 'solution'”. See: <http://home.online.no/~bergar/mazega.htm>

³⁴ “What makes a system self-organized is that the collective patterns and structures arise without the guidance of well-informed leaders, and without any set of predetermined blueprints, recipes or templates to explicitly specify the pattern. Instead, structure is an emergent property of the dynamic interactions among components in the system.”

See: <http://beelab.cas.psu.edu/research/rC/studiesSO.html>

“Any system that takes a form that is not imposed from outside (by walls, machines or forces) can be said to be self-organized. An excellent overview of this question can be found in Francis Heylighen's paper 'The Science of Self-Organization and Adaptivity'”. See:

<http://pespmc1.vub.ac.be/Papers/EOLSS-Self-Organiz.pdf> “.

See, for more details: <http://www.calresco.org/>

Self-organization is basically a process of evolution where the effect of the environment is minimal, i.e. where the development of new, complex structures takes place primarily in and through the system itself. In particular, it is immediately clear for the phenomena of accumulation and order.

For more details: <http://pespmc1.vub.ac.be/SELFORG.html>

³⁵ A combinatory system is ordered, or has a form, if the elements are arranged in an orderly way in a vector or even a multidimensional matrix (cellular automata). Systems with a form can have an emerging macro behaviour for some observers who have specified their point of view.

³⁶ For simplicity's sake such variables are not explicitly included in models [A] and [B].

³⁷ From model [A], we can write, more completely, the model:

$$\begin{array}{l}
 \text{[B]} \quad \left\{ \begin{array}{l}
 \mathbf{mb}(n, t_0) \leftarrow \text{“CHANCE”} \\
 \mathbf{MB}(t_h) = \mathbf{F} \{ \mathbf{MB}(t_{h-1}), \mathbf{C}_{1 \leq n \leq N} [\mathbf{mb}(n, t_h)] \}, \\
 \mathbf{mb}(n, t_{h+1}) = \mathbf{f}_n \{ \mathbf{N}_n [\mathbf{MB}(t_h)] \}
 \end{array} \right.
 \end{array}
 \quad
 \begin{array}{l}
 1 \leq n \leq N \quad \text{[B.1]} \\
 h = 0, 1, 2, \dots \quad \text{[B.2]} \\
 1 \leq n \leq N \quad \text{[B.3]}
 \end{array}$$

The model [B] instead assumes that the *macro behaviour* is determined as well by the past history of the system. Equation [B.2] expresses the *macro behaviour* at a given instant, in part as a function of the macro behaviour of the preceding instant.

In both cases [A.3] and [B.3] represent the fact that the micro behaviour is independent of the micro behaviours from the preceding moments.

³⁸ “Social learning is the phenomenon by which a given agent (the learning agent) updates its own knowledge base (adding to, or removing from it, given information, or modifying an existing representation) by perceiving the positive or negative effects of any given event undergone or actively produced by another agent on a state of the world which the learning agent has as a goal.”. See, for more details on social learning by imitation: Conte – Paolucci [2001]. See:

<http://www.soc.surrey.ac.uk/JASSS/4/1/3.html>

³⁹ There are various definitions of path dependence. The *ingenuous* definition is: “Path dependence is a term that has come into common use in both economics and law. In all instances that path dependence is asserted, the assertion amounts to some version of “history matters”. Path dependence can mean just that: Where we are today is a result of what has happened in the past. For example, the statement “we saved and invested last year and therefore we have assets today” might be more fashionably expressed as, “the capital stock is path dependent.”. See: Liebowitz - Margolis [1998].

The conclusion is “The three types of path dependence make progressively stronger claims. First-degree path dependence is a simple assertion of an intertemporal relationship, with no implied error of prediction or claim of inefficiency. Second-degree path dependence stipulates that intertemporal effects together with imperfect prediction result in actions that are regrettable, though not inefficient. Third-degree path dependence requires not only that the intertemporal effects propagate error, but also that the error was avoidable.”. A more comprehensive definition concerns the dependence of the path of a system on initial conditions. “Sensitivity to initial conditions: when a small change in the initial conditions produces major and unpredictable qualitative changes. Traditional approaches implicitly assume that events occur at an average rate (there are exceptions, and Robust Planning for example does not make that assumption) and that they can be adjusted if they deviate from the desired plan by employing the appropriate adjustment mechanism. But events do not unfold with average regularity and adjustments rarely produce the desired effect. No planning mechanism can take all initial and influencing conditions into account, and at times a small change in the initial conditions produces major and unpredictable qualitative changes. This coupled with positive feedback or increasing returns [Arthur 1990, 1995], makes accurate forecasting and the planning of specific outcomes extremely difficult.”. See: Arthur [1994].

⁴⁰ See: Gleick [1988]

⁴¹ See: Lustick [2000].

⁴² From an exogenous point of view the internal feedback is by definition unobservable (or, in any event, not relevant for the description of the system); thus the science of systems, since it views systems from an *exogenous* point of view, cannot but ignore the internal feedback. See: Sandquist [1985], p. 22.

⁴³ I wish to clearly state that we must not confuse the *endogenous* or *exogenous* point of view with the *external* or *internal* description of a system.

In fact the internal, structural, external, or environmental descriptions always involve systems observed from an *exogenous* point of view; the observer that describes the system does not study the *nature* of the system -- that is, the elements and relations that make up the system and define its state and output; "*The internal description is basically "structural"; that is, it tries to describe the behaviour of the system in terms of state variables and their interdependence. The external description is nevertheless "functional", since it describes the behaviour of the system through its interactions with the external environment.*". See: von Bertalanffy [1968], p. 149.

⁴⁴ The essence of self-organisation is that system structure (at least in part) appears without explicit pressure or constraints from outside the system. In other words, the constraints on form are internal to the system and result from the interactions between the components, whilst being independent of the physical nature of those components. The organisation can evolve either in time or space, can maintain a stable form or can show transient phenomena. General resource flows into or out of the system are permitted, but are not critical to the concept.

For more details, see: <http://psoup.math.wisc.edu/archive/sosfaq.html>

⁴⁵ In the complex systems theory the feedback is considered between agents and not as a determining feature of the system. "*In everyday conversation, we call a system "complex" if it has many components that interact in an interesting way. More formally, we consider a phenomenon in the social, life, physical or decision-making sciences a complex system if it has a significant number of the following characteristics:*

Agent-based: The basic building blocks are the characteristics and activities of the individual agents in the environment under study.

Heterogeneous: These agents differ in important characteristics.

Dynamic: These characteristics change over time, as the agents adapt to their environment, learn from their experiences, or experience natural selection in the regeneration process. The dynamics that describe how the system changes over time are usually nonlinear, sometimes even chaotic. The system is rarely in any long-run equilibrium.

Feedback: These changes are often the result of feedback that the agents receive as a result of their activities.

Organization: Agents are organized into groups or hierarchies. These organizations are often rather structured, and these structures influence how the underlying system evolves over time.

Emergence: The overlying concerns in these models are the macro-level behaviors that emerge from the assumptions about the actions and interactions of the individual agents.".

For more details, see: <http://pacs.physics.lsa.umich.edu/complexity.html>

⁴⁶ The micro-macro feedback is often positive, in the sense that it amplifies the initial casual impulse. At other times it is negative and tends to order the behaviour of the system as a whole by pegging the micro behaviour to the macro behaviour; or, vice-versa, by eliminating the micro behaviours that are deviant with respect to the macro behaviour.

⁴⁷ The global "cooperation" of the elements of a dynamic system which spontaneously emerges when an attractor state is reached is understood as self-organization. Speaking of an attractor makes sense only in relation to its dynamic system; likewise, the attractor landscape defines its corresponding dynamic system.

For details, see: <http://www.c3.lanl.gov/~rocha/ises.html>.

See also: von Foerster [1960]; Haken [1977]; Prigogine [1985]; Kauffman [1993].

⁴⁸ This is the case of populations of insects which act by creating an "aromatic potential field" by spreading *pheromones* or other permanent messages. With their micro behaviours the agents spread pheromone in one site; the increasing concentration of pheromone increases the probability that each agent will move in the direction of that site. The micro-macro feedback is quite evident.

This sequence requires a certain number of insects. Only above a *critical activation mass* of insects can the pheromone amplify and become effective, and lead to some accumulation effect. See: Deneubourg – Goss [1989].

⁴⁹ Currently it is not yet very well understood how the complexity of (cooperative resp. competitive) group behavior is related to individual behavior and how differences in capabilities and problem solution power arise and can be grounded. Even basic concepts such as ... interaction-complexity, communication, organization, minimality etc. are ill- or not uniformly defined:

- *Minimality*: concentrate on simple rather than complex agents, study primitive forms of (spatial) behaviors
- *Collectivity*: takes in principle the group, not the single agent or robot (or animal), as the unit of research analysis and synthesis
- *Locality*: try to fulfill global criteria by exploiting local information

See: di Primio [1999].

See also: <http://ais.gmd.de/~diprimio/bar/workshops/ws4/plain/BAR-Poster-fdp.html>

⁵⁰ In Cellular automata there are many possibilities of defining Neighbourhood. We can mention:

von Neumann Neighbourhood: four cells, the cell above and below and to the right and left of each cell are called the von Neumann neighbourhood of this cell. The *radius* of this definition is 1, as only the next layer is considered.

Moore Neighbourhood: the Moore neighbourhood is an enlargement of the von Neumann neighbourhood containing the diagonal cells too. In this case, the radius $r=1$ too.

Extended Moore Neighbourhood: equivalent to the description of the Moore neighbourhood above, but the neighbourhood extends beyond the distance of the adjacent cells. Hence $r=2$ (or larger).

Margolus Neighbourhood: considers 2×2 the cells of a lattice at the same time.

⁵¹ "It appears to leave human organisations and institutions little different in principle from wasp's nests or even piles of sand. They can all be said to emerge from the actions of the individuals. The difference is that while we assume that, for instance, wasps have no ability to reason - they just go about their business and in doing so construct a nest - people do have the ability to recognise, reason about and react to human institutions, that is, to emergent features. Behaviour which takes into account such emergent features might be called 'second order emergence'. See: Gilbert [1995].

⁵² Except in the case when the micro behaviours are directed by a "director".

⁵³ We must thus remember that in order to produce the micro behaviours (and observe the macro behaviour) we must usually supply energy to the system. Since the main objective of the theory of combinatory systems is to bring out the operative logic typical of such systems, in order to simplify its description energy inputs are usually not considered, in part because such considerations are usually superfluous, if not impossible. In order to give a technical explanation of the action of such systems, in particular for purposes of designing them, the knowledge of the energy inputs can be indispensable.

⁵⁴ We have used, though with a different meaning, the same terminology used by Monod [1971], who, in his famous *Chance and Necessity*, examined a very powerful combinatory system: that leading to a dynamic evolution in a population due to random mutations produced in the DNA that "by necessity" spread as a result of the invariant reproductive mechanism of cells. The author considered "chance" only as the source of the changes in genotype information, and "necessity" only in terms of the reproductive mechanics of the phenotype as caused by the genotype. However, he didn't point out the other features of the combinatory system that describes evolution, and in particular didn't consider the micro-macro feedback action for the spread of new features. Nor did he mention the need for a minimum activation density and, what is more important, didn't include among the necessitating factors the reproductive instinct, and among the recombining factors exogenous teleonomy. Monod writes: "A mutation represents in itself a microscopic, quantum event, to which, as a result, we apply the principle of indetermination: an event which thus is unpredictable by nature." (p. 97). "These alterations are accidental; they occur by chance. And since they represent the sole possible source of changes in the genetic text, which in turn is the sole depository of the organism's hereditary structure, it necessarily follows that chance alone is behind every novelty, every creation in the biosphere. Pure chance, chance alone, absolute but blind freedom, is at the root of the prodigious evolutionary structure: today this central idea of Biology is no longer an hypothesis among the many possible or conceivable hypotheses, but is the only conceivable one, since it is the only one that is compatible with the reality our observation and experience reveal to us." (pp. 95-96).

Haken also speaks of *chance* and *necessity* when he proposes constructing models of complex systems. Here Haken considers *chance* as the unpredictable fluctuation from an unstable equilibrium state, and *necessity* as the movement towards a new, more stable state. Haken [1983]. In this regard, see also Prigogine-Stengers [1993].

⁵⁵ The macro behaviour of many combinatory systems is also characterized by a *direction*. When the macro behaviour can develop with a *direction* or in opposite *directions*, toward one general state rather than another, then this behaviour can be determined by *random fluctuations* in the initial micro behaviours from one to the other state.

"Chance" will not only set under way the macro behaviour but will also determine the *direction*, that is the direction of the "winning" fluctuation. Prigogine bases his theory on the emergence of order in complex systems on the consequences of fluctuations. C.f. Nicolis-Prigogine [1981], passim, in particular section 6 of chapter 6. See also Haken [1983].

A simple way to observe the inflow of the random fluctuations in orientating the *direction* of the "macro" dynamics of combinatory systems - even if it is not sufficient to describe the effect of *chance* on the overall dynamics of a combinatory system - is offered by the so-called *Polya Urns*.

The *Polya urns* : a random elementary process consisting of the random extraction of balls from an urn and the doubling of the extracted ball. A variant of this model is the *Ehrenfest Urns* in which the overall number of balls remains constant but, upon extracting a ball of a certain colour, say "b", we eliminate a ball of another colour. In this case, each random shift in the number of balls influences the probabilities of the succeeding extractions.

⁵⁶ Deneubourg - Goss [1989] show that the task organization in a colony (of bees and/or ants) appears to be a distributed function which does not require a central organizer.

⁵⁷ *Organization* is a typical characteristic of operative systems, but it can also be observed in many natural biological combinatory systems in which the individual elements - for example, cells - can take on different states. Each cell specializes in its own function by taking on the appropriate state in relation to the position it occupies in the system. The form once again appears to guide the system toward a given evolution according to the micro rules contained in the genetic code.

⁵⁸ The characteristic of *ramification* appears in combinatory systems, typically of the diffusion variety, that have a temporal dynamics during which part of the *base* is transformed into another combinatory system that can subsequently expand, and whose elements have certain features in common with the other system and others that are different.

We can thus imagine that a new branch takes off from the original system that in time can be reabsorbed or independently maintained. In the latter case other ramifications can subsequently occur.

There are many systems which permit ramifications: the history of evolution testifies to the progressive ramification of the combinatory system which, beginning with the primordial organisms, has produced the variety of all living species. On the description of the branchings of the living, see for example Maturana-Varela [1987]; the authors refer to branching with the meaningful term *genetic drift*. Monod [1971] considers the combinatory system of evolution and justifies the branchings in terms of random mutations in the genotype and of uniform replications of the new genotype; the resulting phenotype change is spread if it has advantages for life; otherwise it tends to disappear.

⁵⁹ With the *expansion* effect, the micro and macro *rules* that initially operated on a limited base of N elements are extended to an open set of elements.