

INSC 2003
International Nonlinear Sciences Conference
Research and Applications in the Life Sciences
February 7, 8, and 9, 2003
Vienna, Austria

Order and Chaos in Combinatory Systems

A Different Approach to Collective Behaviour

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Abstract

My paper deals with a particular view of synergetics applied to social phenomena produced by collectivities, and aims to demonstrate that collectivities of *non*-interconnected *similar* agents which develop *analogous* micro behaviours can also show very interesting forms of *self-organization* that lead to ordered or chaotic macro behaviour.

I have called these collectivities *combinatory systems* since, on the one hand, the *macro behaviour* of the system as a whole derives from the *combination* of the analogous *micro behaviours* or *effects* of the agents and, on the other, the *macro behaviour* determines, conditions, or directs the subsequent micro behaviours. This internal *micro-macro feedback* produces a *self-organization* effect as if an Invisible Hand or Internal Organizer regulated its time path and produced the observable effects and patterns.

Combinatory systems are not easily recognizable; nevertheless they are widely diffused and produce most of the social and economic collective phenomena involving the accumulation of objects, the spread of features or information, the pursuit of a limit, and the achievement of general progress as the consequence of the individual pursuit of particular interests.

My second aim is to illustrate – with the aid of simple *combinatory automata* – phenomena as intriguing as they are emblematic: the voice-noise effect in organizations; the clustering and swarming effects in economics; the unjustified raising of retail prices; the stock exchange dynamics deriving from the micro-macro feedback between stockbroker decisions and the stock index. We will see that the joint action of crossed and multi-level micro-macro feedback makes it not unthinkable that “a butterfly can cause the collapse of the stock exchange”.

Keywords: behaviour of collectivities; combinatory systems; combinatory automaton; synergetic; social dynamics.

1 – Collectivities

Definition 1 – I define a collectivity as a plurality of similar elements or agents which are unorganized – that is, not specialized according to function, functionality, functioning and topology – and produce an analogous micro behaviour over time – or similar micro effects – but, considered together, are capable of developing a macro behaviour– and/or macro effects – which is attributed to the collectivity as a whole¹.

Collectivities can be *observable* (for example, swarms, flocks, crowds, spectators at a stadium, students in a classroom, persons that are talking in a crowded room, dancers doing the Can Can), or simply *imaginable* (for example, trailer-trucks traveling a stretch of highway in a month, the noble families of Pavia who erected the 100 towers in the span of two centuries, a group of scientists who dedicate themselves to a branch of research, the consumers of a particular product during its entire life-cycle, stockbrokers working on a certain day in world or European stock markets).

Collectivities can be composed of persons (social collectivities or populations), animals active biological organisms (herds, schools, swarms, etc.), or reactive ones (plants, micro-organisms), even other inanimate objects; in any case, if considered from a certain distance collectivities appear distinct with respect to the individuals they are composed of, and thus seem able to show an autonomous macro behaviour due to the joint action of the micro behaviours of the agents.

The behaviour of the collectivity can be defined as *local* – or based on *limited information* – or *global* – or based on *complete information* – depending on whether or not the macro behaviour derives from local information possessed by the agents (a person acquires a good because he observes that at least N friends have bought it; an elephant in a herd runs to the left because the elephants on its right push it in this direction) or from global information (over time and/or space) possessed by all the agents (all the students rise because the teacher orders them to; all the animals flee because they see the fire advancing)².

Global information plays an essential role in many relevant phenomena produced by social collectivities.

Global information may derive from outside (external director, starting traffic lights or the starting gun in races, trumpeting by the leader of the herd, and so on) or may be self-produced by the joint micro behaviours of the agents considered as a whole.

I define social collectivities acting on the basis of self-produced information as Combinatory Systems.

I observe that if, on the one hand, it is easy to explain (perhaps properly speaking, to describe), assuming only local information and rules, the behaviour of a flock of birds, a school of fish, or a herd of elephants when these collectivities have already formed, or the spread of information, the imitation of choices (information contagion), or the percolation effects in probabilistic diffusion

¹ A collectivity of *organized agents* forms an organization, that is a social system (a social machine according to Maturana and Varela, 1980) where the collective and individual behaviour is determined by a network of stable relations (that is, the organization).

² In Agent-Based Models collectivities are normally interpreted as Complex (Adaptive) Systems (Coveney and Highfield, 1995; Mitleton and Kelly, 1997; Allen, 1997; Axelrod, 1997; Goldspink, 2000), defined as a plurality (usually large) of blind (reactive) or intelligent (active) multi-character (Drogoul and Ferber, 1994), specialized, usually (strongly) interconnected (Wu, 1997; Granovetter, 1974; Grimmett, 1999) interacting agents (or processes) (Holland, 1995; Gell-Mann, 1995-96; Stacey, 1995), often showing possible multi-level hierarchies (Chan, 1998; Gaffeo, 1999; Cummings and Staw, 1985: 2) whose collective macro behaviour is determined by the interaction of the micro behaviours of the agents (Otter, Veen and Vriend, 2001) on the basis of simple local rules (Waldrop, 1993) according to a schema (innate or learned) (di Primio, 1999), and which shows non-linear dynamics as well as unanticipated global properties, or patterns (Foster and Metcalfe, 2001: 4).

systems (Frey and Decker, 1996; Grimmet, 1999), on the other hand it is not so easy to apply this micro approach to describe, for example, the grouping of flocks (a bird is attracted by the flock and not by its neighbours), swarms, herds and other collectivities, the formation of graffiti on walls (people are attracted by the cloud of graffiti and not by the behaviour of other people), the breaking out of applause (many people applaud if the applause dies down), or the phenomenon of a rising murmur in a crowded room.

It is clear that a person who is talking raises his voice to go beyond the increasing murmur of the crowded room only for individual necessity, and not because his neighbours are raising their voices, or that a fish joins a school of fish because of the presence of a predator, and only if he can perceive the school, and not because he sees other fish join the school.

Similarly, it is hard to explain, by exclusively using local rules, the exceeding of limits (all people park or drive fast even in the presence of parking limits and speed limits), the pursuit of records, the eternal maintenance of feuds, and the phenomenon of urban settlements.

The analysis and understanding of these and many other phenomena, which will be mentioned below, is even more difficult because they often are "one way" and cannot be repeated or reproduced, as if due to chance.

While the phenomenon of urban settlements appears to repeat itself many times, even with particular variations, the same cannot be said for the construction of towers in medieval Pavia, which is an amazing event because it is unique.

In many cases, moreover, Agents cannot observe the collectivity, and thus their neighbours, and must act only based on individual necessities, as in the case of the formation of piles of garbage (if I need to throw away a piece of garbage and I see a garbage pile, I prefer to leave my garbage behind), of annoying and dangerous wheel ruts on the highway (passing trucks need to maintain their trajectory on the carriageways, and this is reinforced by these micro behaviours), or of paths in fields (people prefer to cross a field where a path is visible), and so on.

In all these circumstances, the Agents' micro behaviours seem to follow some *necessitating global self produced information* represented (or derived) by some macro variable(s) deriving from the collectivity (the cloud of graffiti, the pile of garbage, the applause, the carriageway, the feud, and so on) rather than obey a set of local rules or information².

² The Complex Adaptive Systems approach, in particular (Allen 1997), studies how collectivities interact and exchange information with their environment to maintain their internal processes over time through adaptation, self preservation, evolution and cognition (in the sense of Maturana and Varela 1980: 13), and to achieve collective decisions (Rao and Georgeff 1992: 127-146, Wooldridge and Jennings 1994) within a relational context of micro behaviours (Conte and Castelfranchi 1992).

The analysis of complex systems implies a Recursive Approach, and two of the most powerful tools are represented by the Cellular Automata Theory – introduced in the late 1940s by John von Neumann (Burks 1966), which allows the researcher to explore complex systems by simulating Artificial Life (Alife) (Liekens 2000) – and the Genetic Algorithms approach (Bak 1994, 1996).

The theory of Cellular Automata builds mathematical models of a system whose agents are represented by cells in an array (a lattice) of one or more dimensions (Creutz 1996, Schatten 1999). It is important to note that the rules that define the micro behaviour of a cell are only local rules, in the sense that the state of the cell depends only on one of a specified number of neighbours and not on the state of the array (Gardner 1970, Toffoli and Margolus 1987, Dewdney 1989, 1990, Ulam 1986, 1991).

Following the logic of cellular automata, many fundamental instruments have been created to simulate Artificial Societies (Resnick 1994, Epstein and Axtell 1996, <http://zooland.alife.org>). Among the most well-known are Dorigo's *Ants* approach (Dorigo, Di Caro and Gambardella 1999, Hölldobler and Wilson 1990), Langton's *Swarm* approach (<http://www.swarm.org>), Reynolds's boids (Reynolds 1987), and Dolan's *Floys* approach (Dolan 1998).

These instruments also demonstrate that there is also a hidden order in the behaviour of collectivities of simple living autonomous reactive agents.

As Holland attempts to demonstrate, the most powerful approach to understanding and showing the hidden order in collective behaviour is the genetic algorithms approach (Holland 1975) and the related genetic programming approach of Koza (Goldberg 1989, Koza 1992).

2 – Combinatory Automaton

Definition 2 – I define as (mono-dimensional) Combinatory Automaton a particular automaton whose macro dynamics is regulated by a combination (to be specified) of the micro dynamics of its cells, according to the following specifications (Fig. 1):

1) let there be a set of N cells A_i , $1 \leq i \leq N$, arranged in a **combinatory lattice** A ; each cell represents an agent;

2) all the agents are characterized by the variables a_i – which are similar because they are defined in the same domain (or repertoire) $d_i = d \subset \mathbf{R}$ – whose values represent the state $a_i(t_h)$ at time $t_h \in T$ (we assume T is a discrete time scale);

3) the **analytical state** of the automaton, $A(t_h) = [a_i(t_h)]$, is defined as the values $a_i(t_h) \in d_i$ assumed by A_i for each $t_h \in T$;

4) in many cases a set of functions f_i transforms each $a_i(t_h)$ into a different variable $e_i(t_h) = f_i[a_i(t_h)]$ that may represent the **output micro effect** produced by agent A_i , $1 \leq i \leq N$; in most cases, we may assume $e_i(t_h) = a_i(t_h)$; the **time series** $A_i(T) = [e_i(t_0), e_i(t_1), e_i(t_2), \dots]$ represents the **micro behaviour** of the agent A_i over a period T ;

5) the **synthetic state** of the automaton at t_h is defined as the value assumed by a global macro variable $X(A, t_h) = \{C_{1 \leq i \leq N} a_i(t_h)\} = C[A(t_h)]$ derived from a **combination** of those values, where $C_{1 \leq i \leq N}$ indicates a set of **combination operation(s)**, appropriately specified (sum, product, average, min, max, etc.), of values associated with $A(t_h)$;

6) the **output macro effect** of the automaton at t_h is defined as the value assumed by the variable, $E(A, t_h) = F \{X(A, t_h)\} = F \{C[A(t_h)]\}$; the **recombining function** F (or **macro rule**) transforms the **synthetic state** into the **output of the automaton**; the **time series** $E(A, T) = [E(A, t_0), E(A, t_1), E(A, t_2) \dots]$ represents the **macro behaviour** of the automaton in period T ; in many simple cases, $E(A, t_h) = X(A, t_h) = C[A(t_h)]$;

7) at time t_{h+1} each A_i changes its value following the **micro transition function**: $a_i(t_{h+1}) = N_i[a_i(t_h), X(A, t_h)]$, where N_i represents the **necessitating function**, that is a set of **necessitating operation(s)**, appropriately specified (difference or variation), which modifies the previous values $a_i(t_h)$ following the **output variable**, $X(A, t_h)$; in many simple cases, $a_i(t_{h+1}) = a_i + k_i [a_i(t_h) - X(A, t_h)]$ (Melay automata) or $a_i(t_{h+1}) = K_i [X(A, t_h)]$ (Moore automata) where k_i and K_i represent appropriate scalars; the transition of state is characterized by a length, Δt_i , and may follow a probability, p_i (par. 3);

8) for the recursive dynamics being produced, we must also assume that the **initial state** $A(t_0)$ is specified;

9) as a result, a general **micro-macro feedback** relation connects the micro state to the macro state through the **variable** $E(A, t_h)$, which may be thought of as an **organizing** or **driving variable** of the corresponding **combinatory system**, since it determines the subsequent micro behaviour of the agent A_i ;

10) the **set of rules** specifying the initial state $A(t_0)$, the operations $C_{1 \leq i \leq N}$ and N_i , the rules F and f_i , and the probabilities p_i , represent the operative **programme**, which produces the dynamics of the **combinatory automaton**.

The definition is summarized in the following formal model (Fig. 1):

| | | | | |
|-----|---|--------------------|--------------------------|-------|
| | $A(t_0) = [a_i(t_0)] \leftarrow \text{CHANCE/PROGRAMME}$ | $1 \leq i \leq N$ | Initial analytical state | [A.1] |
| | $X(A, t_h) = \mathbf{C}_{1 \leq i \leq N} [a_i(t_h)] = \mathbf{C} [A(t_h)]$ | $h=0, 1, 2, \dots$ | Synthetic state | [A.2] |
| [A] | $E(A, t_h) = \mathbf{F} \{ X(A, t_h) \}$ | | Output macro behaviour | [A.3] |
| | $a_i(t_{h+1}) = \mathbf{N}_i [a_i(t_h), p_i, \Delta t_i, E(A, t_h)]$ | | Analytical state | [A.4] |
| | $e_i(t_{h+1}) = f_i \{ \mathbf{N}_i [a_i(t_h), p_i, \Delta t_i, X(A, t_h)] \}$ | $1 \leq i \leq N$ | Output micro behaviours | [A.5] |
| | Set: $\{ \mathbf{C}_{1 \leq i \leq N}, \mathbf{N}_i, p_i, \Delta t_i, \mathbf{F}$ and $f_i \}$ | | Operative programme | [A.6] |

3 – Typology of Combinatory Automata

Considering the *modus operandi* of the combinatory automaton, it is useful to point out that a combinatory automaton may be:

- *stochastic*, if a probability p_i is associated with the transition of state of each A_i , $1 \leq i \leq N$; in the opposite case it is *deterministic*; probabilities may be: *fixed* if $p_i \equiv p$ for every i and j ; *time dependent* if $p_i \equiv p(t_h)$; *time and agent dependent* if $p_i \equiv p_i(t_h)$; *output dependent* if $p_i \equiv p_i(X, t_h)$;
- *time-response sensitive*, if the length of the *period of transition of state* $\Delta t_i = t_{i+1} - t_i$ is agent-output dependent; $\Delta t_i \equiv \Delta t_i(X, t_h)$;
- *two-dimensional*, if Agents are arranged in R rows and C columns, so that $N = (R * C)$, or *multi-dimensional* (in the model I have considered a mono-dimensional automaton);
- *mono* or *multiple-driven*, depending on the number of driving variables $E_j(A, t_h)$ (in the model I have considered a mono-driven automaton and $j=1$ is omitted);
- *reversible*, if $a_i(t_h) = a_i(t_k)$, $h \neq k$, is admitted (in the model I have not explicitly considered reversibility, as this will be developed in the next section);
- *casual* or *causal*, if, respectively, the initial state, $A(t_0)$, is produced *by chance* or is attributable to a specific *activation programme* of an external will.

In *stochastic combinatory automata*, when both *probabilities* $p_i \equiv p_i(X, t_h)$ and *periods* of transition of state $\Delta t_i \equiv \Delta t_i(X, t_h)$ are agent/time/state sensitive, the *micro behaviours* are conditioned by the *macro behaviour* of the entire system, which makes the *micro-macro feedback* evident.

Probabilities can act in two ways:

- as *stop-or-go probabilities*, in the sense that if the probabilistic event occurs, the agent assumes a new state; otherwise, it maintains its actual state. We might symbolize this type of probability by writing: $p_i(X, t_h)_{|0,1|}$. "0" means that if the event does not occur, the agent maintains its state; "1" that the agent changes its state if the event occurs;
- as *transition probabilities*, in the sense that if the probabilistic event occurs, then the agent enters a new state; if the event does not occur, the agent assumes a different state or returns to the past one. We can write: $p_i(X, t_h)_{|-1,1|}$.

With regard to the nature of the *combination operation* carried out, we can define a combinatory automaton as:

- *medial combinatory automaton*, if its *synthetic state* and *output* at t_h are defined as a function of the mean (to be specified) of the values assumed by the micro variables, for example: $X(A, t_h) = \sum_{1 \leq i \leq N} a_i(t_h)$; the subsequent *micro transition function* may be based on the deviation from

the mean value or be a function of the mean value; in the simplest case, for example: $\mathbf{a}_i(\mathbf{t}_{h+1}) = \mathbf{N}_i [\mathbf{a}_i(\mathbf{t}_h) - X(\mathcal{A}, \mathbf{t}_h)] = \mathbf{N}_i [\Delta \mathbf{a}_i(\mathbf{t}_h)]$, for any cell \mathbf{A}_i , $1 \leq i \leq N$;

- *maximal combinatory automaton*, if its **synthetic state** and **output** at \mathbf{t}_h are defined as a function of the maximum value in the analytical state; for example: $X(\mathcal{A}, \mathbf{t}_h) = \mathbf{Max}_i \mathbf{a}_i(\mathbf{t}_h) = \mathbf{a}_M(\mathbf{t}_h)$; consequently, $\Delta \mathbf{a}_i(\mathbf{t}_h) = \mathbf{a}_i(\mathbf{t}_h) - \mathbf{a}_M(\mathbf{t}_h)$ represents the *quantum of inferiority* perceived by each \mathbf{A}_i , $1 \leq i \leq N$ compared with the state of $\mathbf{a}_M(\mathbf{t}_h)$ which can be interpreted as the *leader cell* (or “the best”);

minimal combinatory automaton, if $X(\mathcal{A}, \mathbf{t}_h)$ is a function of the minimum value in the analytical state; in the simplest case, for example: $X(\mathcal{A}, \mathbf{t}_h) = \mathbf{Min}_i \mathbf{a}_i(\mathbf{t}_h) = \mathbf{a}_m(\mathbf{t}_h)$; in this case, $\Delta \mathbf{a}_i(\mathbf{t}_h) = \mathbf{a}_i(\mathbf{t}_h) - \mathbf{a}_m(\mathbf{t}_h)$ represents the *quantum of superiority* perceived by each \mathbf{A}_i , $1 \leq i \leq N$ compared with the state of $\mathbf{a}_m(\mathbf{t}_h)$, which can be interpreted as the *base cell* (or “the worst”).

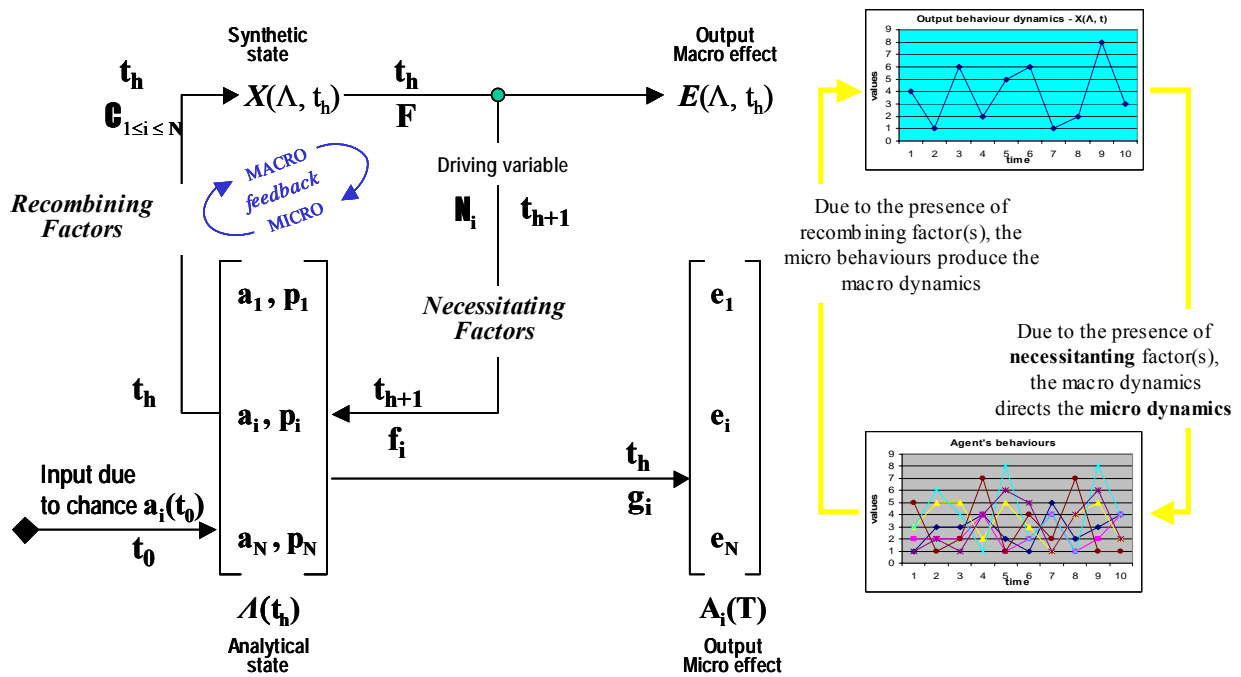


Figure 1 – The operative logic of a combinatory automaton

4 – Combinatory Systems

Definition 3 (operational) - I define as a combinatory system any collectivity whose behaviour may be simulated by a combinatory automaton.

This definition is quite simple but focuses on the operative logic of any combinatory system: each cell corresponds to an Agent; the lattice corresponds to the Combinatory system as a whole; on the one hand, the macro behaviour of the System, as a whole, derives from the combination (defined in an opportune way) of the analogous micro behaviours of its similar agents (hence the name Combinatory System) but, on the other hand, the subsequent micro behaviours derive, in some way (are determined, or conditioned, or directed) by the macro behaviour of the system.

Combinatory systems are thus characterized by a *micro-macro feedback* between the micro and macro behaviours.

We can suppose that a necessary and sufficient condition for a collectivity (observable or supposable) to be considered a combinatory system is the existence of a feedback between the micro behaviour of the individuals and the macro behaviour of the collectivity constituting the

system.

The feedback arises from *necessitating factors*, which force the agents to adapt their micro behaviour to the system's macro behaviour, and is maintained by the action of *recombining factors*, which lead the collectivity to recombine the *micro* behaviours, or the micro effects, in order to produce and maintain the macro behaviour, or the macro effect.

Recognizing *the existence of a micro-macro feedback and understanding the nature of both the necessitating factors and the recombining ones is indispensable for interpreting collective phenomena as deriving from a combinatory system*³.

In this sense *path dependence* (Arthur 1988, Liebowitz and Margolis 1998) is proof of the action of the micro-macro feedback, even if path dependence theory does not explicitly include this mechanism in the explanation of the path dependence.

Definition 4 (cognitive) - *I define as a combinatory system any collectivity whose agents, consciously or unconsciously, act (exclusively or prevalently) on the basis of global information which they directly produce and update as the consequence of their micro behaviours. On the one hand, the global information is - or derive from - a synthetic variable whose values derive from the combination of the micro states of the agents but, on the other, these values affect the subsequent states as a result of a micro macro feedback, acting over a period, that produces self-organization in the agents' micro behaviours* (Fig. 2).

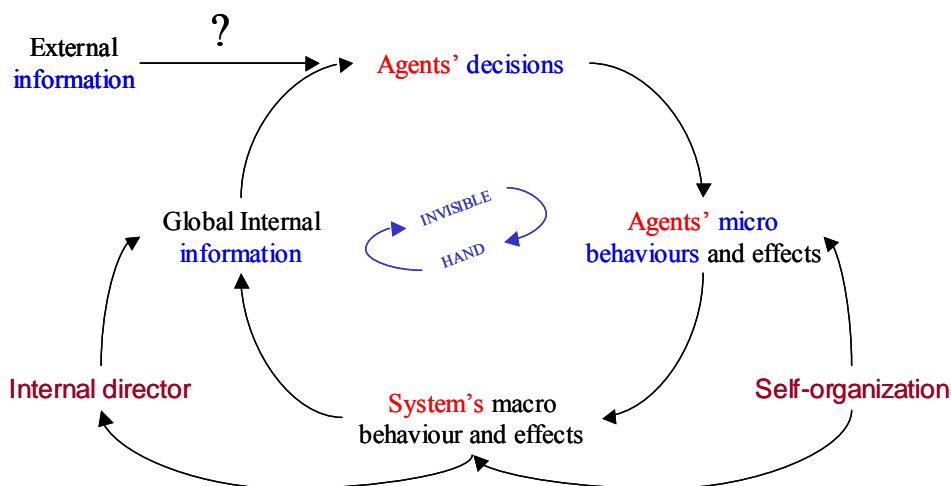


Figure 2 - The cognitive logic of a combinatory system

This definition emphasizes the cognitive activity of the agents: the macro effects produced by the macro behaviour of the system in themselves do not necessarily lead to self-organization; they become factors in self-organization only when these effects are interpreted by the agents as information they can base their decisions on.

Examples: if an immigrant (or a new entrepreneur) looks at an urban (industrial) settlement (macro effect) he may argue that the settlement offers better conditions of life (of business) than elsewhere (global information derived from the macro effect: the impetus belli, the applause (see below), the accumulation of pheromones, are macro effects (moving into attack, clapping, ant colony columns) that can be interpreted as global information for other agents (the attack has begun and I must follow, the performance is finishing and I must appreciate it, the path is reliable and I can follow it).

³ In order to provide a technical explanation of the action of such systems, and above all for the purpose of planning them, knowledge of the energy inputs can turn out to be indispensable.

Since by definition the agents are similar and have similar behaviour, it follows that we can assume that the same information produces similar decisions regarding the change in state of the agents, who thus appear to conform or even *synchronize* their micro behaviours.

The typical example of evident synchronization is that of applause. How many times have we experienced this!

A certain number of persons attend an event. Suddenly someone – *by chance* or *directed* by someone – claps (micro behaviour), thereby producing a typical sound (micro effect). If the number of those that begin to clap does not reach the *minimum activation number*, then the applause does not begin. But if the initial clapping does not *die down*, others will join in and there is thundering applause. The micro behaviours translate into a macro behaviour (everyone applauding), of which the applause, understood as a typical sound, represents the macro effect and the global information according to which the subsequent micro behaviours are synchronized.

The individual spectators produce the applause by clapping their hands, but this obliges everyone *by necessity* to continue to clap their hands in order to sustain the applause itself until someone stops applauding and the macro effect fades away. The feedback inevitably acts in the opposite direction, and the applause slowly dies out.

Combining the two previous definitions, if we accept the traditional notion of self-organization as the macro behaviour of a collectivity of agents in which the micro behaviours appear to be directed, or organized, by an Invisible Hand, or Supreme Authority, or Benevolent Deity in order to produce the emerging phenomenon represented by the formation of ordered structures, of recognizable patterns (Foster and Metcalfe 2001: 130, Pelikan 2001), then all the above-mentioned collective phenomena can also be defined as self-organization or spontaneous order (Sugden 1989, Kauffman 1993, Ashford 1999, Swenson 2000)³.

There is nothing strange here: the *invisible hand* is nothing other than the *synergetic effect* of the *micro-macro feedback* action (or *circular causality*) that generates and updates the *global information* that produces *self-organization* and emerging macro behaviours attributable to the collectivity⁴.

The micro-macro feedback may be thought of as a *internal dynamic director* or, better yet, as an *internal dynamic organizer* which produces and uses the *global information* as an *order parameter*⁴ and, following the *slaving principle*, directs or organizes the individual behaviours and

³ Adam Smith's invisible hand naturally comes to mind. Adam Smith used the term "invisible hand" only once in his *Wealth of Nations* (1776) in the following quotation: "...[B]y directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. Nor is it always the worse for the society that it was not part of it."

The invisible hand was also mentioned by Haken, the founder of Synergetics: "We find that the various parts are arranged as if guided by an invisible hand and, on the other hand, it is the individual systems themselves that in turn create this invisible hand by means of the coordinated effect. We shall call this invisible hand that gives order to everything the «organizer» (Haken, 1977).

⁴ This is the case of populations of insects, typically ants, which act by creating an "aromatic potential field" by spreading *pheromones* or other permanent messages. With their micro behaviours the agents spread pheromone across one site (micro information); the increasing concentration of pheromone (global or macro information) increases the probability that each agent will move in the direction of that site. The micro-macro feedback is quite evident (Zollo, Iandoli and De Maio 2001). This behaviour is the consequence of *stigmergy*, which derives from the ability of ants to communicate by means of small signals able to trigger a chain reaction (Grassé 1959).

This sequence requires a certain number of insects. Only above a *critical activation mass* of insects can the pheromone amplify and become effective, and lead to some accumulation effect (Deneubourg and Goss 1989).

⁴ When an ordering parameter guides system components or subsystems, this is said to *slave* the subsystems, and this *slaving principle* is the key to understanding self-organizing systems. The global information produced by a combinatorial system may be considered an ordering parameter that slaves all the agents of the system and forces them to self-organize and produce collective phenomena.

"In general just a few collective modes become unstable and serve as "ordering parameters" which describe the macroscopic pattern. At the same time the macroscopic variables, i.e., the order parameters, govern the behavior of the

produces the self-organization of the system and hence the collective phenomena (von Foerster 1960, Haken 1977, Prigogine 1985, Kauffman 1993, Martelli 1999)⁵.

5 – Social Combinatory Systems

The most interesting combinatory systems are the social ones, which are made up of men or organizations.

On the basis of the considerations in the previous sections, we can define social combinatory systems by the following definition, which nevertheless can also be applied to more general cases.

Definition 5 (functional) - *I define as a (social) combinatory system any collectivity showing the following functioning rules (Fig. 3):*

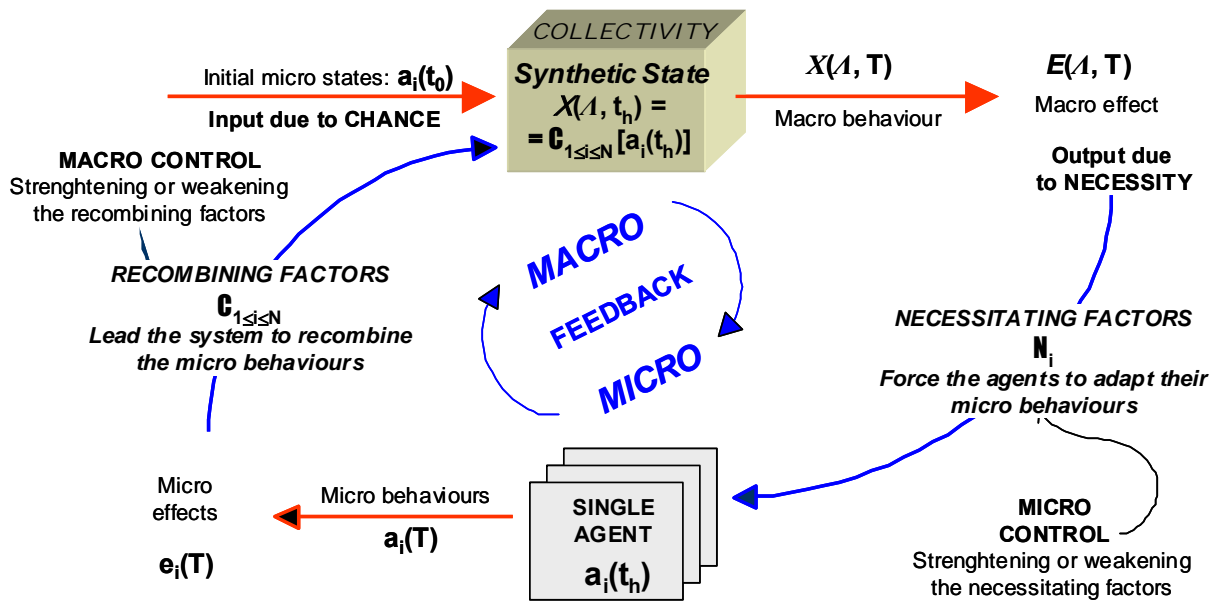
- all agents are similar in the sense they show a relatively similar nature, structure or significance;
- these are not necessarily interconnected by evident interactions, or by network, web or tree structures;
- all the agents are characterized by the same *individual variable* (or set of variables) of some kind (qualitative or quantitative) whose values – at any time t_h – represent the *individual states* whose dynamics, over a period T, may be defined as *micro behaviours* of the agent – which may lead to analogous *micro effects*;
- the collectivity is characterized by a *macro (global) variable* (qualitative or quantitative) whose values – at any time t_h – represent the *system states* whose dynamics over a period T may be defined as a *macro behaviour* (which may lead to a *macro effect* of some kind) attributable to the collectivity as a whole;
- the system state – at any time t_h – derives from the *combination* (to be specified) of the individual states, following macro or *recombining rules*, and may be conceived – or interpreted – as a *global information* for the agents; in many cases the global information corresponds to the macro behaviour or the macro effect of the collectivity as a whole;
- each agent – at time t_{h+1} – through the *global information*, can perceive and evaluate – in a simple pay-off table – positive or negative gaps (advantages or disadvantages) between his individual state and the state of the collectivity; following the micro or *necessitating rules* each agent makes individual micro decisions (by a process of imitation and social learning) in order to increase (if positive) or reduce (if negative) the perceived gaps;
- but these decisions recursively change the value assumed by the *macro variable*, and this modifies the perceived positive or negative gaps, driving the agents to adapt their behaviour by new decisions.

microscopic parts by the “slaving principle. In this way, the occurrence of order parameters and their ability to enslave allows the system to find its own structure”. (Haken, 1988: 13).

The *micro-macro feedback* is the expression of the *circular causality* which emerges when the subsystems collectively determine the order parameters and the order parameters determine the behavior of the subsystems.

⁵ *“In general just a few collective modes become unstable and serve as “ordering parameters” which describe the macroscopic pattern. At the same time the macroscopic variables, i.e., the order parameters, govern the behavior of the microscopic parts by the “slaving principle. In this way, the occurrence of order parameters and their ability to enslave allows the system to find its own structure.”.* (Haken, 1988: 13) *“In general, the behavior of the total system is governed by only a few order parameters that prescribe the newly evolving order of the system”* (Haken (1987: 425).

Figure 3 - The main elements of a combinatory system



The *operative logic* of combinatory systems is as basic as their structure:

- on the one hand the micro behaviours seem self-organized to produce the macro behaviour of the system which, for an observer, may be conceived as an *emergent phenomenon*;
- on the other hand, the macro behaviour updates the global information and determines, conditions, directs, or drives the subsequent micro behaviours in a typical micro-macro feedback; this, for an observer, may be conceived as a *self-organization effect*;
- the micro-macro feedback operates between the limits of the *minimum activation number* and the *maximum saturation number* of the agents presenting the state that maintains the micro-macro feedback; this guarantees over time both the production of the *emergent phenomenon* and the maintenance of the *self-organization effect*.

6 – Typology of Combinatory Systems

The logic proposed in the previous sections can be observed in five relevant classes of combinatory systems which differ with regard to their macro behaviour (or their macro effect):

1. Systems of accumulation, whose macro behaviour leads to a macro effect which is perceived as the accumulation or the clustering of “objects”, behaviours, or effects of some kind; this logic applies to quite a diverse range of phenomena, among which the formation of urban or industrial settlements of the same kind and of industrial districts, the grouping of stores of the same type in the same street, the accumulation of garbage, graffiti, writings on walls; but it can also be applied to phenomena such as the breaking out of applause, the formation and the maintenance of colonies, forests, herds and schools.

2. Systems of diffusion, whose macro effect is the diffusion of a trait or particularity, or of a “behaviour” or “state”, from a limited number to a higher number of agents of the system; systems of diffusion explain quite a diverse range of phenomena: from the spread of a fashion to that of epidemics and drugs; from the appearance of monuments of the same type in the same place (the towers of Pavia, for example) to the spread and maintenance of a mother tongue, or of customs.

3. Systems of pursuit produce a behaviour that consists in a gradual shifting of the system toward an “objective”, as if the system, as a single entity, were pursuing a goal or trying to move toward increasingly more advanced states; this model can represent a lot of different combinatory systems: from the pursuit of records of all kinds to the formation of a buzzing in crowded locales; from the start of feuds and tribal wars in all ages to the overcoming of various types of limits.

4. Systems of order, produce a macro behaviour, or a macro effect, perceived as the attainment and maintenance of an ordered arrangement among the agents that form the system; systems of order can be used to interpret a large number of phenomena: from the spontaneous formation of ordered dynamics (for an observer) in crowded places (dance halls, pools, city streets, etc.) to that of groups that proceed in a united manner (herds in flight, flocks of birds, crowds, etc.); from the creation of paths in fields, of wheel-ruts on paved roads, of successions of holes in unpaved roads, to the ordered, and often artificial, arrangement of individuals (stadium wave, Can-Can dancers, Macedonian phalanx).

5. Systems of improvement and progress, whose effect is to produce progress, understood as an improvement in the overall state of a collectivity that is attained through individual improvement. This can be considered a subclass of the Systems of Pursuit. Individual improvements raise the parameter that measures collective progress; this leads to the formation of positive and negative gaps that push the individuals to improve in order to increase the gaps (if positive) or eliminate them (if negative). The system must be able to perceive the individual improvement and to adjust the progress parameter to the average (or, more generally, to the combination) of the individual improvement measures.

7 – Probabilistic social combinatory systems

The *social combinatory systems* that are most interesting and easiest to represent are the *irreversible* ones (build a tower or not, teach Italian or English to babies). In these systems both the micro and macro behaviours produce permanent effects that may be viewed as increasing or decreasing cumulative processes in which the probabilities are: $\mathbf{p}_i(\mathbf{X}, \mathbf{t}_h)_{[0,1]}$.

Chaos arises in *combinatory systems* when the *hypothesis of reversibility* is introduced (for example: to speak or to keep quiet in the next minute, wear a skirt or miniskirt on different days, choose road A or B on different days) (Fuchs and Haken 1989). These systems are generally governed by *transition* probabilities: $\mathbf{p}_i(\mathbf{X}, \mathbf{t}_h)_{[-1,1]}$.

When reversibility in micro behaviours or in micro effects is possible, the combinatory system’s macro behaviour, or macro effect, can show a cyclical dynamic and, under certain conditions concerning the probability function regarding the transition of state of the elements, a chaotic one as well, when no cycles are recognizable in the time series of the system starting from random initial values (Gleick 1988). Examples of reversible systems are those of diffusion and dissemination (fashion and contagion), whose elements may at different times present the same state chosen from a repertoire (Lustick 2000).

In particular we can note that in probabilistic reversible combinatory systems both the random initial states of the system and the probability function for the transition of states, which depend on the macro behaviour at each iteration, can be determined with ample approximation.

These hypotheses of randomness in the initial conditions and in their evolution as well (*history dependence*), together with the imprecision of the measurement of the micro behaviours, produce dynamic instability in the macro behaviour and explain almost all the cases of path dependence, both in reversible and, in many cases, irreversible systems, as we can argue from [A.1] in the previous models (Liebowitz and Margolis 1998, Arthur 1988, 1994).

8 – A Combinatory Automaton simulating Improvement and Progress

A very special and important combinatory system is the one I have named the *Improvement and Progress Combinatory System*, since its particular effect is to produce progress, according to commonly accepted value judgements regarding an improvement in the overall state of a collectivity⁵.

The systems of IMPROVEMENT AND PROGRESS may be simulated by a Combinatory Automaton following these very simple rules based on the general definition in section 2:

- 1) the *analytical state* of the automaton, $\mathcal{A}(\mathbf{t}_h)=[\mu_i(\mathbf{t}_h)]$ is defined as the values of the parameter of improvement $\mu_i(\mathbf{t}_h)$ associated with each \mathbf{A}_i in $\mathbf{t}_h \in \mathbf{T}$; the succession $[\mu_i(\mathbf{t}_0), \dots, \mu_i(\mathbf{t}_h), \dots]$ represents the *improvement path* of \mathbf{A}_i ;
- 2) the *synthetic state* and the *output* of the automaton at \mathbf{t}_h are defined as the value assumed by the *macro variable* $\pi(\mathcal{A}, \mathbf{t}_h) = (1/N) \sum_{1 \leq i \leq N} \mu_i(\mathbf{t}_h)$, which represents the parameter of progress for the entire system; the succession $[\pi(\mathcal{A}, \mathbf{t}_0), \dots, \pi(\mathcal{A}, \mathbf{t}_h), \dots]$ represents the *progress path* of $\mathcal{A}(\mathbf{t}_h)$;
- 3) at time \mathbf{t}_h the *necessitating operation(s)*, which condition the internal event (decision) that determines the agents' behaviour, derives from the difference $\mathbf{N}_i = \Delta\mu_i(\mathbf{t}_h) = \mu_i(\mathbf{t}_h) - \pi(\mathcal{A}, \mathbf{t}_h)$, which denotes the *deviation* between the *individual improvement* level and the mean level denoting *collective progress*; so that each \mathbf{A}_i perceives an *inferiority*, with respect to the mean, if $\Delta\mu(\mathbf{n}, \mathbf{t}) < 0$, or a *superiority* in the opposite case, and acts to maintain or increase the advantage, or eliminate or reduce the disadvantage;
- 4) the *micro transition* functions can assume the following expression; $\mu_i(\mathbf{t}_{h+1}) = \{ \mu_i(\mathbf{t}_h) + \mathbf{p}_i \mathbf{i}_i \Delta\mu_i(\mathbf{t}_h) \}$; the probabilities $\mathbf{p}_i = \mathbf{p}(\Delta)_{[0,1]}$ (the probability is not only agent-dependent but is assumed to be dependent, for each agent, on the sign of $\Delta\mu_i(\mathbf{t}_h)$) represent the *necessitating factors* under the hypothesis of irreversibility (agents can only ameliorate their improvement measures), or $\mathbf{p}_i = \mathbf{p}(\Delta)_{[-1,1]}$ under the hypothesis of reversible micro behaviour (agents may also reduce their improvement measures with respect to the progress measure); the parameter \mathbf{i}_i indicates the random action of \mathbf{A}_i to ameliorate his performance;
- 5) to make the model more general, I have also supposed that the *micro transition* functions which determine the change in the agents' behaviour depend also on *environmental factors*, that is on *external events*, whose general expression is: $\{ \mathbf{r}_i [\mathbf{k} \mu_i(\mathbf{t}_h) + \mathbf{h} \pi(\mathcal{A}, \mathbf{t}_h)] \}$, where $\mathbf{r}(\Delta)_{[0,1]} = \mathbf{r}_i$ (or $\mathbf{r}(\Delta)_{[-1,1]} = \mathbf{r}_i$ in the case of reversibility) indicates the probability of this external event –

⁵ The economy considered as a typical combinatory system of improvement and progress was theorized firstly by Adam Smith; he asserted that each individual strives to become wealthy, *considering only his own gain*, but to this end he must produce with the maximum productivity and exchange what he owns or produces with others who are capable of evaluating the goods he has to offer. The human desire to improve wealth and individual happiness produces progress in general wealth and welfare; Smith argued that the division of labour and a free market are the *recombining factors* necessary for the system to operate. The maximization of returns on invested capital represents the *necessitating factors* together with the necessity of happiness as well. "Every individual is continually exerting himself to find out the most advantageous employment of whatever capital he can command. It is his own advantage, indeed, and not that of the society, which he has in view. But the study of his own advantage naturally, or rather necessarily leads him to prefer that employment which is most advantageous to society." (Smith 1776).

Also Ludwig von Mises clearly described the economic system (consumers and producers) as an improvement and progress combinatory system. "The rich adopt novelties [improvement] and become accustomed to their use [producing positive gaps]. This sets a fashion which others imitate [mean value increases]. Once the richer classes have adopted a certain way of living, producers have an incentive to improve the methods of manufacture [recombining factor] so that soon it is possible for the poorer classes to follow suit [reducing negative gaps]. Thus luxury furthers progress. Innovation "is the whim of an elite before it becomes a need of the public. The luxury today is the necessity of tomorrow." [recombining factor]. Luxury is the roadmaker of progress [necessitating factor]: it develops latent needs and makes people discontented [recombining factor]. In so far as they think consistently, moralists who condemn luxury must recommend the comparatively desireless existence of the wild life roaming in the woods as the ultimate ideal of civilized life." (Ludwig von Mises 1981) [square brackets are ours]

assumed to be dependent, for each agent, on the sign of $\Delta\mu_i(t_h)$ – and $[k \mu_i(t_h) + h \pi(A, t_h)]$ represents the amount of influence of the environmental variables on the improvement measure of A_i . This expression translates the common idea that the attempt to improve performance is conditioned by both the previous level of the individual performance measure and the previous level of performance of the system (k and h are scalar coefficients, but we may normally assume that $h=0$).

- 6) the micro and macro dynamics are thus connected, since the level of improvement measures determine the level of progress, but this in turn modifies the subsequent improvement variables in the typical *micro-macro feedback*;

The combinatory automaton is summarized in the formal model:

$$\begin{array}{l}
 \text{[B]} \left\{ \begin{array}{ll}
 \mu_i(t_0) \leftarrow \text{“CHANCE”} & 1 \leq i \leq N \quad \text{[B-1]} \\
 \pi(A, t_h) = (1/N) \sum_{1 \leq i \leq N} \mu_i(t_h) = X(A, t_h) & h=0, 1, 2, \dots \quad \text{[B-2]} \\
 \mu_i(t_{h+1}) = \{ \mu_i(t_h) + p_i i \Delta\mu_i(t_h) \} + \{ r_i [k \mu_i(t_h) + h \pi(A, t_h)] \} & 1 \leq i \leq N \quad \text{[B-3]} \\
 \Delta\mu_i(t_h) = \mu_i(t_h) - \pi(A, t_h) = N_i & 1 \leq i \leq N \quad \text{[B-4]}
 \end{array} \right.
 \end{array}$$

and, due to the structure in [B-2], we can name it the **MEDIAL AUTOMATON OF IMPROVEMENT AND PROGRESS**.

Two important remarks need to be made.

At first I observe that, even if the **MEDIAL AUTOMATON OF IMPROVEMENT AND PROGRESS** is quite general, we may conceive of two other different automata simply by specifying equation [B-2].

1. Assuming:

$$\pi(A, t_h) = \text{Max}_i \mu_i(t_h) = \mu_M(t_h)$$

we have defined the *Maximal Automaton of Improvement and Progress* (or “of *pursuit*”); consequently, in equation [B-4],

$$\Delta\mu_i(t_h) = \mu_i(t_h) - \mu_M(t_h)$$

represents the *quantum of inferiority* perceived by each agent compared with the improvement parameter of the *leader agent* (or “the best”). We thus witness micro behaviours aimed at reducing the inferiority with respect to the level of progress, and this causes a macro behaviour whose effect is to raise the average level of improvement, so that some agents manage to further raise the previous level of progress.

2. Assuming:

$$\pi(A, t_h) = \text{Min}_i \mu_i(t_h) = \mu_m(t_h)$$

we have defined the *Minimal Automaton of Improvement and Progress* (or “of *flight*”); in equation [B-4]:

$$\Delta\mu_i(t_h) = \mu_i(t_h) - \mu_m(t_h)$$

represents the *quantum of superiority* perceived by each agent compared with the improvement parameter of the *base agent*.

These systems act in a symmetrical way with respect to the previous ones, since each agent of the system tries to outdistance as much as possible its own level of improvement from the level of progress, to flee from the minimum level of improvement, and to increment its own superiority.

This leads to a general increase in the average level of improvement, which ends up raising the parameter of progress, further boosting the levels of improvement.

Secondly, let us note that if both p_i and r_i admit reversibility, then the system is *strong reversible*; if only one of the two probabilities admits reversibility (generally r_i), the system is *weak reversible*; elsewhere it is *irreversible* and improvement and progress are continuously increasing.

Let us assume an automaton of ten agents described by Figure 4, which also shows the dynamics of this system under different hypotheses of reversibility.

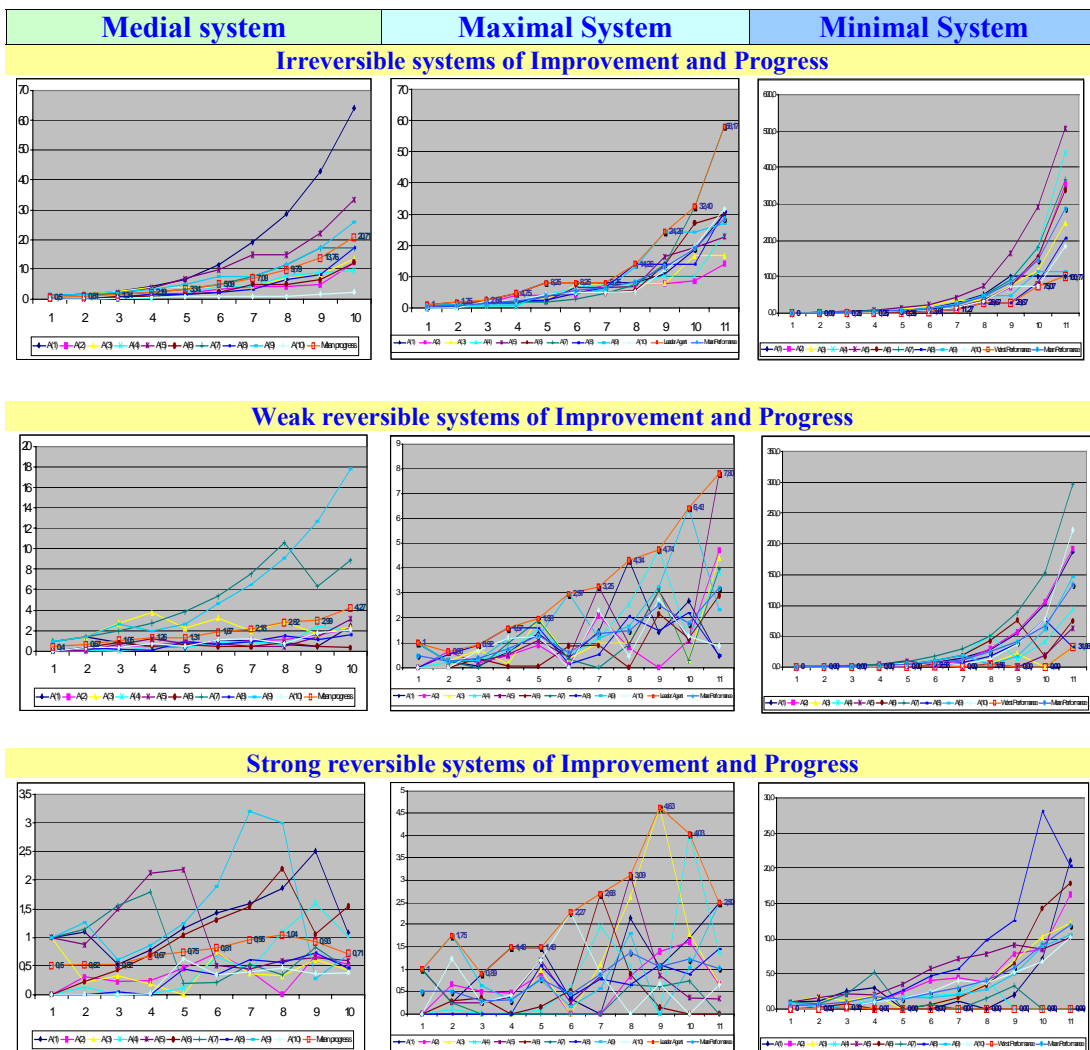
Figure 4 – Three types of systems of improvement and progress

Table 1 – Improvement and Progress Combinatory Systems (10 agents, 10 iterations)

1.(A) – Data

| Probabilities ↓ | A(1) | A(2) | A(3) | A(4) | A(5) | A(6) | A(7) | A(8) | A(9) | A(10) |
|------------------------|---------|------|------|------|------|------|------|------|------|-------|
| $p_A(n, t) = p_A(n) =$ | 20% | 20% | 20% | 20% | 20% | 20% | 20% | 20% | 20% | 20% |
| $p_A(n, t) = p_A(n) =$ | 80% | 80% | 80% | 80% | 80% | 80% | 80% | 80% | 80% | 80% |
| $r_A(n, t) = r_A(n) =$ | 80% | 80% | 80% | 80% | 80% | 80% | 80% | 80% | 80% | 80% |
| $r_A(n, t) = r_A(n) =$ | 20% | 20% | 20% | 20% | 20% | 20% | 20% | 20% | 20% | 20% |
| Initial states | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| Parameters | k = 0,5 | | 0,5 | | = h | | | | | |

1.(B) – Simulations Simulations (red lines indicates the progress variable; coloured lines indicate agents' improvement variable)



As we can easily note, the more reversibility is introduced, the more the macro and micro behaviours are chaotic, as we can verify by simulating dynamics for 20 iterations. Among the phenomena that can be explained using the system of improvement and progress are the growth of productivity in firms, the continuous improvement in the quality of products, progress in the sciences and in technology, and the evolution of all types of species as a consequence of individual choices.

9 – A Combinatory Social System producing the Voice and Noise effect in crowded rooms.

Let us now consider the phenomenon of a murmur arising in a crowded room, typically produced by a *combinatory system* of talking people. The murmur is the output of the crowded room considered as a combinatory automaton and is produced by the combination of the voice levels of the individual speakers who, in order to make themselves heard, must raise their voices some decibels above the murmur. But recursively this increases the murmur, in a typical feedback between micro and macro behaviour.

For an observer, the talking agents thus seem self-organized to simultaneously raise their voice level and produce a stable, a rising, or a fluctuating noise, a typical pattern which, I am sure, we have all experienced several times.

We can represent this phenomenon through the *medial stochastic combinatory automaton* [C]⁶:

$$[C] \begin{cases} A(t_0) = v_i(t_0) \leftarrow \text{“CHANCE”} & 1 \leq i \leq N \\ \mathbf{G}_{1 \leq i \leq N} [v_i(t_h)] = (1/N) \sum_{1 \leq i \leq N} v_i(t_h) \\ \mathbf{M}(A, t_h) = \{ k [(1/N) \sum_{1 \leq i \leq N} v_i(t_h)] + Q r(t_h)_{[0,1]} \} (1 - a) & h = 0, 1, 2, \dots \\ v_i(t_{h+1}) = \{ [w_i \mathbf{M}(A, t_h) + v_{i(\min)}] + v_{i(\text{rnd})} l_i(t_h)_{[0,1]} \} s_{i [0,1]} \mathbf{b}_i(\text{bol}) (t_h); & 1 \leq i \leq N \end{cases}$$

The simulation model of Figure 5 shows this phenomenon; it describes a linear *stochastic medial automaton* of 20 (non-ordered) speakers observed for 30 iterations.

The voice levels (colored lines) are the variables associated with the speaking agents.

The noise (bold blue line) may be viewed as the output of the combinatory automaton constituting the collectivity considered as a whole.

The crowded room recombines the voice levels into a simple mean $[(1/N) \sum_{1 \leq i \leq N} v_i(t_h)]$, but the level of noise also depends on several factors – the nature of the speakers, the necessity of speaking, the structure of the room that, recombining the voices, can maintain or reduce the murmur – which specify a set of appropriate parameters for the macro and micro functions **F** and **f**.

In particular, the *necessity to speak* is represented by a probability that may or may not depend on time and on the number of talking people. If we introduce tolerance into the model, that is the maximum level of bearing, then the system may show a cyclical behaviour.

10 – Medial combinatory systems producing a growth in prices

This combinatory system is very common and operates whenever a certain number of operators, A_i , involved in the retail markets of a certain good, decide to purchase or sell at certain prices, taking into account the *global information* represented by the *price index* for that and/or other goods. The

⁶ For details, see: <http://www.ea2000.it/cst/CSTteory/CST-SO.pdf>. All the models may be algebraically rearranged and simplified.

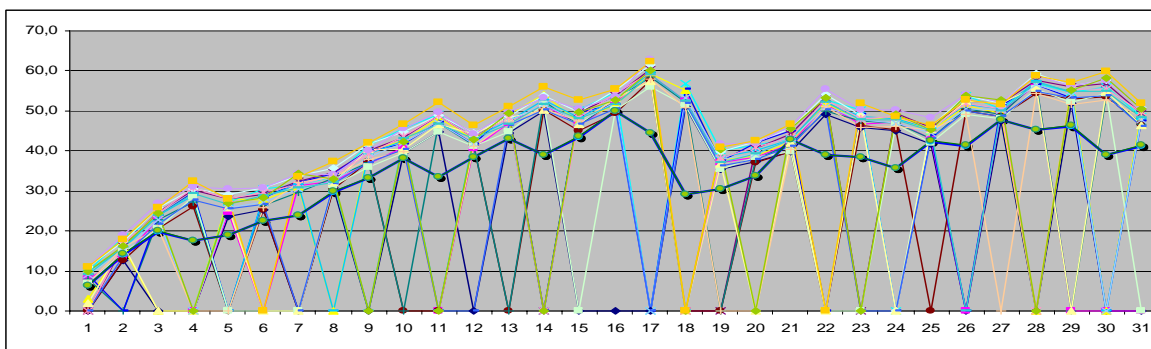
price index $P(t_h)$ is represented by the weighted average of prices $p_i(t_h)$ that exists for every A_i but at t_{h+1} each A_i adjusts its own prices by taking into account the gap represented by $[P(t_h) - p_i(t_h)]$. The system is simulated by a *medial combinatory stochastic automaton*.

The simulation model in Figure 6 shows this phenomenon; it describes a *mono-dimensional medial stochastic automaton* of 10 (non-ordered) operators observed for 20 iterations.

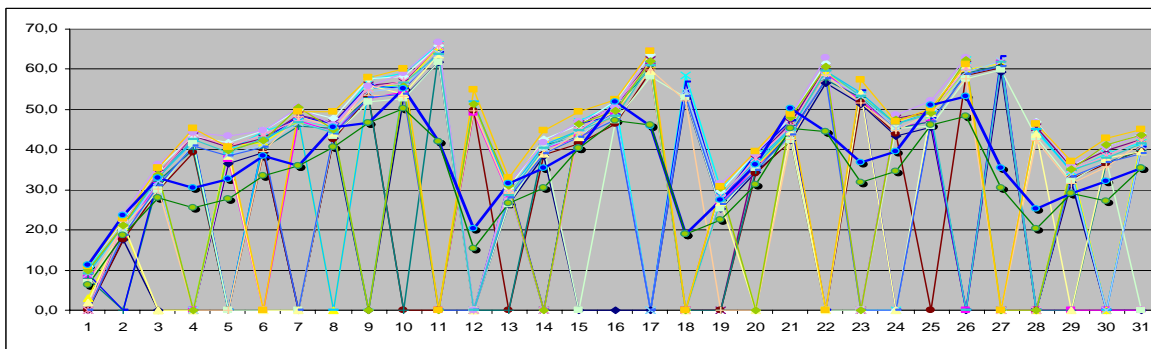
The bold blue line represents the global information, that is the price index; the coloured lines show the micro behaviour of the prices quoted by each operator. The red line indicates the trend growth of the price index.

Figure 5 - Model of Murmur and Noise system with 20 agents and differentiated probabilities for each Agent

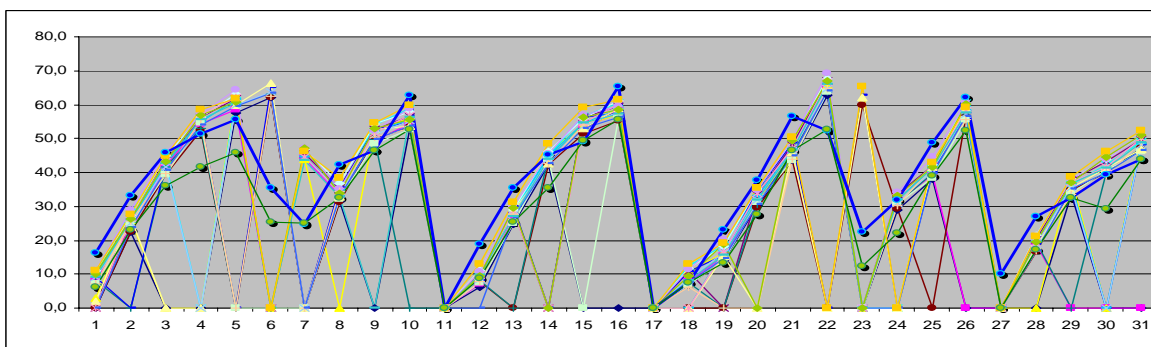
Test 1 – External noise $Q = 0$ decibels. Mean probability to speak = 80%



Test 2 – External noise $Q = 10$ decibels (all other parameters equal)



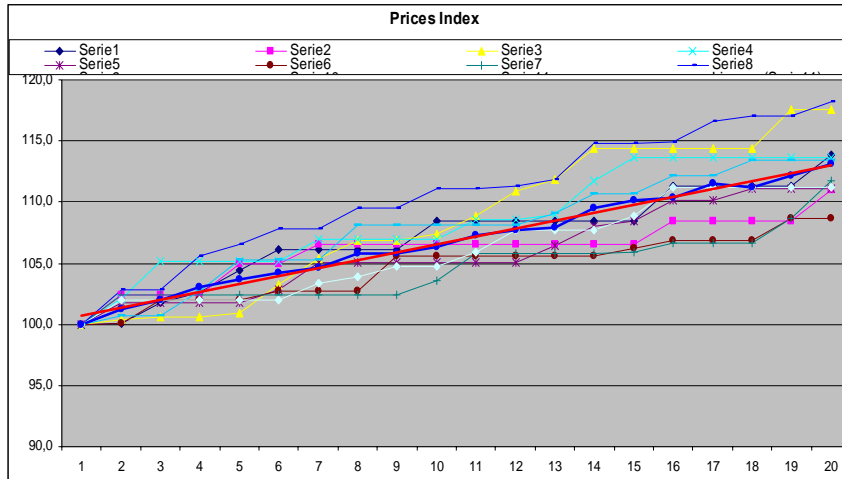
Test 3 – Mean probability to speak = 85% (all other parameters equal)



A similar though much more complex combinatory system is represented by the stock exchange, which produces chaotic dynamics in the quotations of individual stocks and in the general indices.

Let us consider the simpler case of a certain number of stockbrokers interested in trading a certain stock, and let us suppose that their purchase or selling decisions are taken only considering the price index $S(t_h)$ for that stock.

Figure 6 - Model of Increasing price system with 10 agents and differentiated probabilities for each Agent



We can easily see that if an operator A_i at time t_h possesses a stock with a value $s_i(t_h) < S(t_h)$, the potential gain is equal to the difference; thus A_i can decide to keep the stock, perhaps hoping for a further rise, or sell it for a gain. If instead $s_i(t_h) > S(t_h)$, then there is a potential loss equal to the difference. A_i can thus decide to keep the stock, hoping perhaps for a rise in value, or sell it to limit the loss. To simulate the behaviour of the operators by means of a *stochastic combinatory automaton* we must introduce selling probabilities that can be differentiated based on the sign and level of the difference: $\Delta s_i(t_h) = [s_i(t_h) - S(t_h)]$. Once the decisions are made the new prices, which depend on the level of $S(t_h)$ as well as on random factors, lead to changes in the stock quotation, which becomes $S(t_{h+1})$ and will influence subsequent selling decisions. For simplicity's sake the purchasing decisions are not explicitly considered, but are held to be linked to the selling ones.

Since the behaviour of each agent depends only on the information regarding the gap between the value of the stock that is held and the stock quotation of the share, this automaton is only based on *one-level feedback* between micro and macro behaviours, as indicated in figure 7. This model specifies that the stockbrokers make decisions not only taking account of the level of $S(t_h)$ but also other exogenous information.

In fact, as we all know, trading decisions regarding a stock also depend on the general level of stock exchange quotations, which derives from the weighted average of the quotations of the various stocks (Fig. 8). If the general quotation represents further information for an individual stockbroker, then the stock exchange also becomes a combinatory system where the brokers decide based on two levels of feedback, one represented by the quotation of their own stock and the second by the general stock exchange index, as shown in the model in figure 9.

Nevertheless the behaviour of stockbrokers depends not only on the individual stock quotation but also that for other shares. In this case we can define the combinatory system as a *crossed-feedback combinatory system* (Fig. 10).

Figure 7 - Model of price dynamics produced by a the combinatory automaton simulating a single stock with 10 agents and differentiated probabilities for each Agent

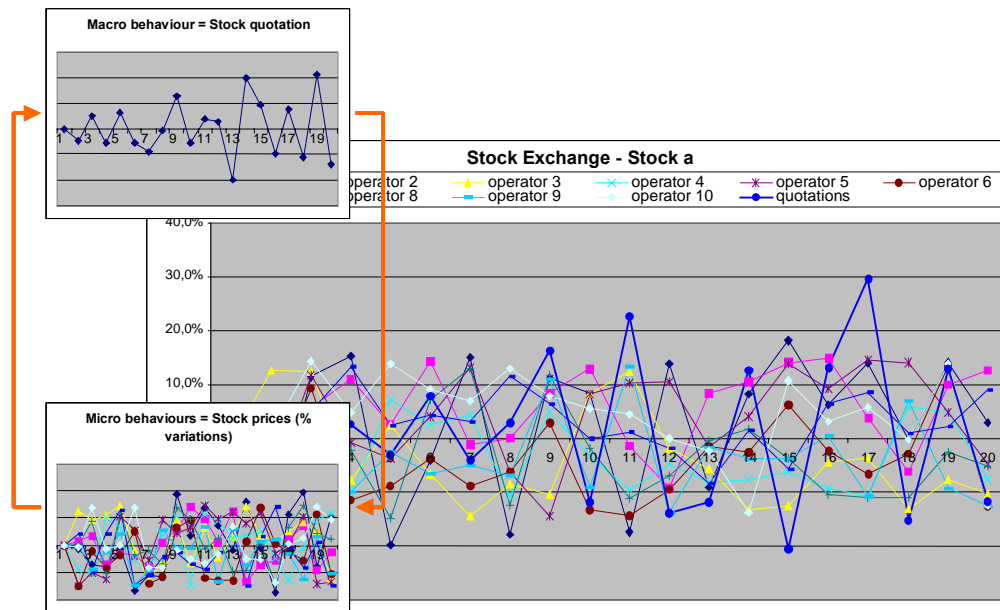
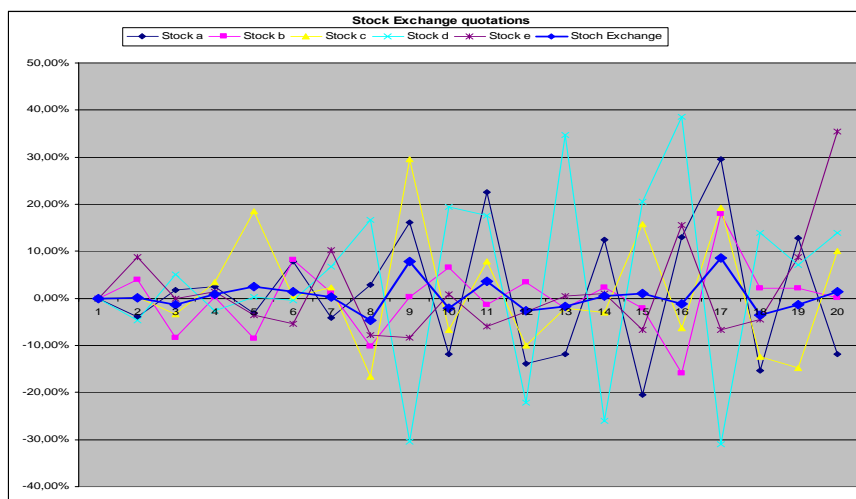


Figure 8 - Model of a Stock Exchange in which 5 stocks are quoted



We obtain an important conclusion from these models: the general price index of a stock exchange is the result of a complex combinatory system, holonic in nature and composed of complex combinatory systems interrelated by means of multiple and crossed feedback. The interconnected stock exchanges are similar to a holarchy, which is a *hierarchically organized structure of holons* (Fig. 11)⁶.

⁶ Koestler's *holonic systems* approach represents a different approach with respect to Agent-Based Systems (Koestler 1968; Shimizu 1987; Wilber 2000), particularly useful for studying the behaviour of living organisms and social organizations. These are composed of self-reliant units that are capable of flexible behaviour. More specifically though, a holon can be thought of as a special type of agent that is characteristically autonomous, cooperative and recursive, and that populates a system where there is no high-level distinction between hardware and software. Holons form Holarchies, defined as *hierarchically organized structures of holons*. In a Holarchy each Holon could be regarded as either a whole or as a part, depending on how one looks at it. A Holon will look as a whole to those parts

Figure 9 - Model of the combinatory system of a single stock (a) quotations showing one-level feedback.

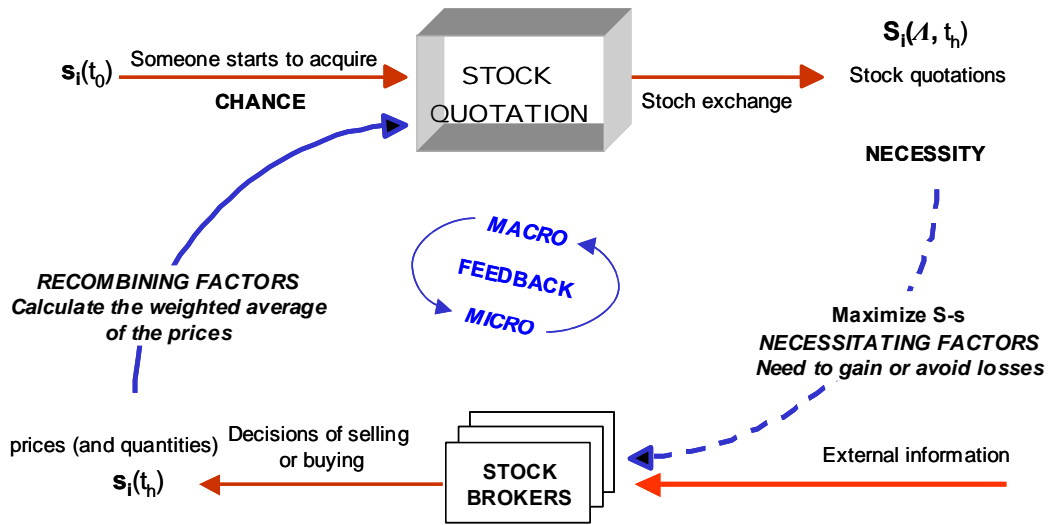
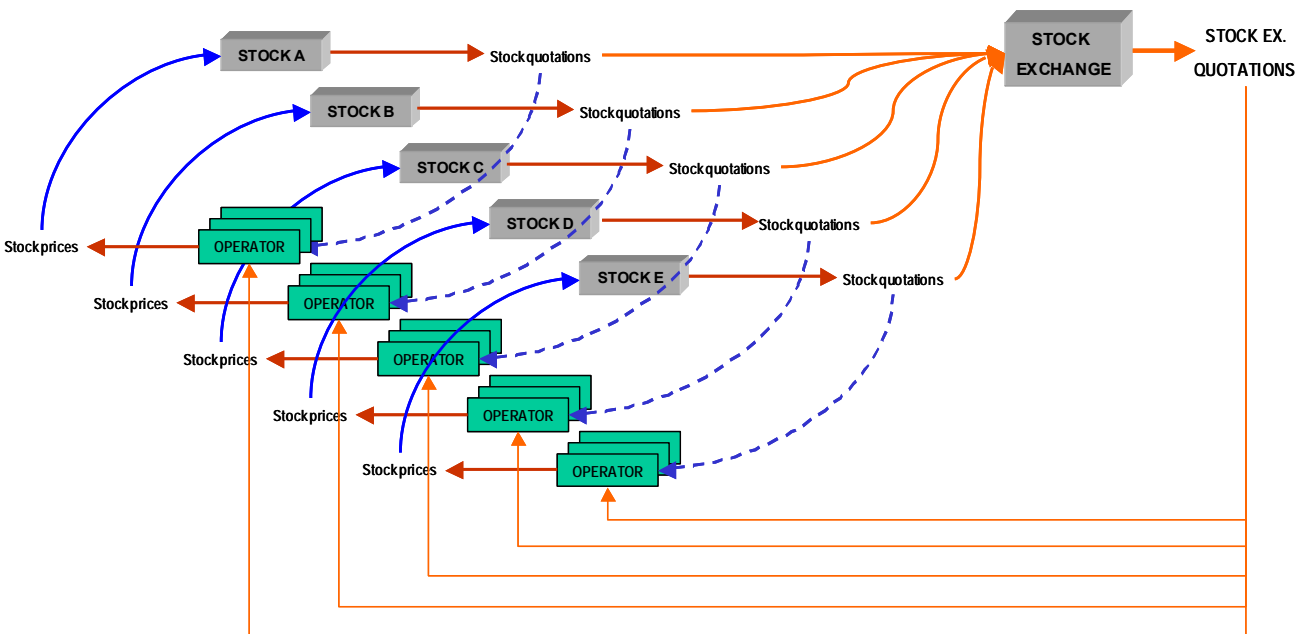


Figure 10 - Model of the combinatory system showing two-level feedback, and crossed feedback which determines the formation of the Stock Exchange quotations

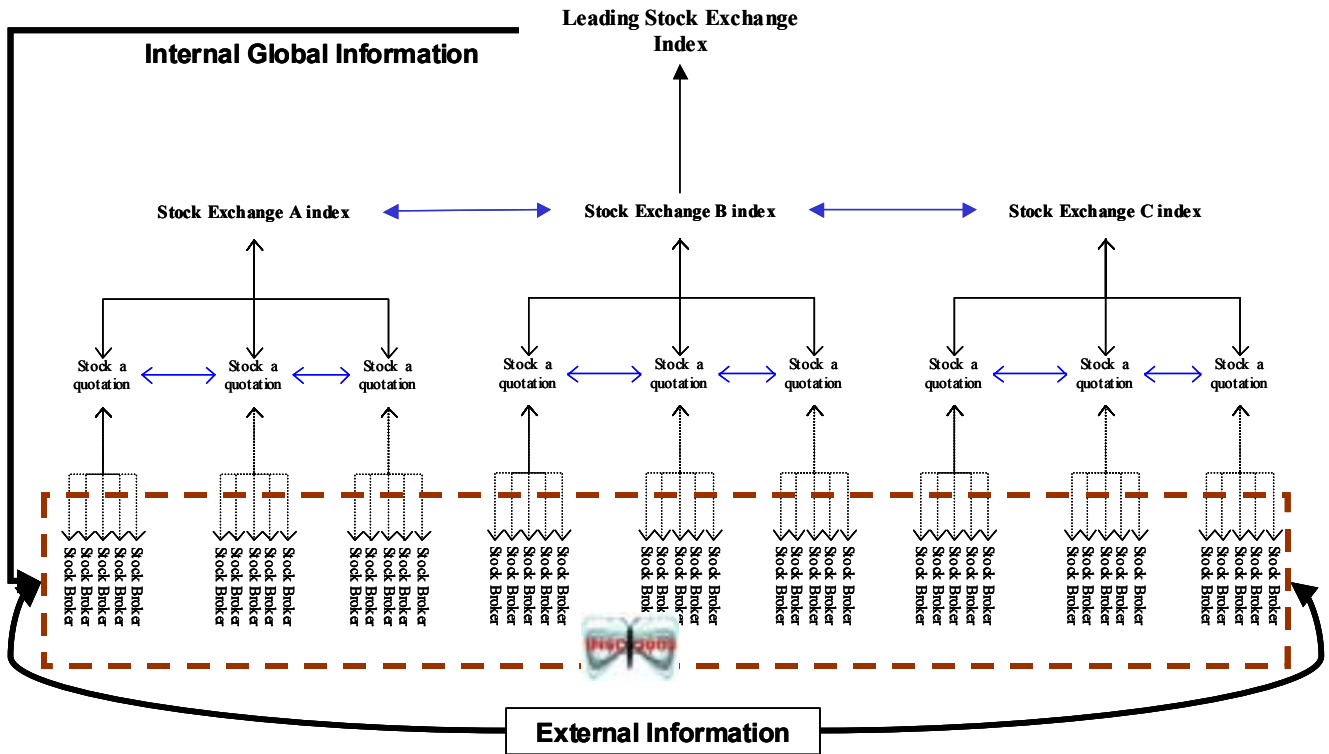


It is possible that the broker's decision to sell a stock can, in a certain sense, set off a combinatory dynamic process for that stock. The dynamics regarding that stock can then influence that of other stocks, in the same as well as opposite direction. The combination of the dynamic processes for these stocks determines that of the general index, which in turn influences stockbrokers' decisions by modifying these decisions, or strengthening or inverting the direction of the trend. Finally, the world exchanges are interconnected by the feedback process so as to form a single combinatory system.

beneath it in the hierarchy, but it will look as a part to the wholes above it. Thus a Hierarchy is a whole that is also a structure of parts that are in themselves wholes.

It is thus not difficult to imagine how the beating of a butterfly's wings can condition some operators and have catastrophic results for the dynamics of a stock exchange, and that this can condition the dynamics of the entire system of world exchanges.

Figure 11 – Model of the holonic structure of the Stock Exchange network



11 – Races and Records as Combinatory Systems

A *competition* – for example, a car race – can be viewed as the effect of a maximal combinatorial system. The racers are agents who produce a position in the race; the global information is represented by the standings at every instant of the race. The leader considers it necessary to maintain his lead over the followers; those following the leader find it necessary to reduce the gap. Thus the racers try to continually adjust their states, thereby producing micro behaviours in order to gain and maintain the highest positions. This combinatorial system is *reversible* in that the individual racers can modify their position during the race (Fig. 12).

We can also assume the irreversibility of the results gained by those participating in the competition, as occurs, for example, in all attempts at breaking a record.

A *record*, in any sports category, determines the "absolute best"; because of this we witness a true race for the record.

Those who compete are not content to equal the record, but instead do all they can to beat it. Thus, records are gradually improved and represent at one particular moment the macro effect of the combinatorial system of the athletes and the global information that directs the subsequent micro behaviours.

The attempts to improve the records motivate more and more athletes to take part in competitions, which also leads to the continual improvement in the *average performance* of the athletes, so that after a more or less lengthy period, beginning when the first record was set, the average performance of the competitors is very high.

12 – Combinatory systems of accumulation and of diffusion

We can define as *accumulation systems* those combinatory systems whose macro behaviour leads to a macro effect that can be perceived as an accumulation of objects, types of behaviour, or effects of some kind.

If we have to accumulate some object with others similar in nature (micro behaviour), we normally look for already-made accumulations, since this gives us an advantage or reduces some disadvantage (necessitating factor).

The accumulation represents the global information that directs the choices of the Agents to accumulate or disperse the objects.

If the environment preserves the accumulated objects or is not able to eliminate them, and maintains the advantages of the accumulation, this favours the accumulation (macro behaviour), and an accumulation of some kind is created (macro effect). The micro-macro feedback is evident: the larger the accumulation (macro effect), the more incentive there is to accumulate (micro behaviours) objects (micro effects); the collective accumulation (macro behaviour) leads to an ever greater accumulation.

These systems are normally irreversible.

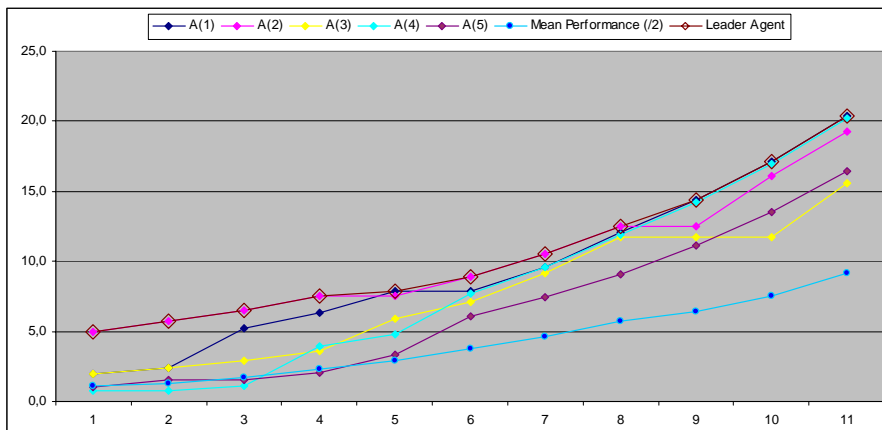
Another very important and diffused class of combinatory system is represented by the combinatory systems that lead to the spread of objects or to the diffusion of a feature, a peculiarity, or a "state" from a limited number to a high number of elements of the system.

The global information is represented by the observed or hypothesized diffusion of objects or features among the collectivity.

A greater diffusion (macro effect) implies a greater desire to acquire the object (micro effect); the single acquisition (micro behaviour) widens the collective diffusion (macro behaviour).

Many systems of this kind are reversible, and reversible choices may imply complex micro and macro behaviours.

Figure 12 – Combinatory maximal system of pursuit simulating a race with 5 pursuers acting over 10 iterations



13 – Path Dependence and Chaos in Reversible Combinatory Automata Simulating Diffusion

According to the general model [A] of section 2 it is easy to build the following *Combinatory Automaton* simulating *diffusion* and *accumulation*:

| | | | | |
|-----|--|---|---------------------|--------------------------------|
| | A_i for which $\mathbf{a}_i(\mathbf{t}_0) = \{\mathbf{0} \text{ or } \mathbf{1}\}$ | $1 \leq i \leq 50$ | Agents | |
| [D] | { | $A(\mathbf{t}_0) = [\mathbf{a}_i(\mathbf{t}_0)] \leftarrow \text{“CHANCE”}$ | | Initial analytical state [D.1] |
| | | $X(A, \mathbf{t}_h) = \sum_{1 \leq i \leq N} \mathbf{a}_i(\mathbf{t}_h) = N(\mathbf{t}_h);$ | $h=0, 1, 2,$... | Synthetic state [D.2] |
| | | $\mathbf{a}_i(\mathbf{t}_{h+1}) = \mathbf{f}_i \{ \mathbf{M}_i[\mathbf{a}_i(\mathbf{t}_h) \mathbf{p}_i[N(\mathbf{t}_h)]] \};$ | $1 \leq i \leq 50$ | Transition of state [D.3] |
| | | Define $\mathbf{p}_i[N(\mathbf{t}_h)]$ | | Operative Programme [D.4] |

Let us first consider a Combinatory Automaton simulating diffusion.

In the previous point 6) of the general model [A] of the *Combinatory Automaton*, we can simply assume that $\mathbf{a}_i(\mathbf{t}_h) = \text{“0”}$ “or” $\mathbf{a}_i(\mathbf{t}_h) = \text{“1”}$ according to a probability of transition of state; so we simply write:

$$\mathbf{a}_i(\mathbf{t}_{h+1}) = \{\text{“1” } \mathbf{p}_i[N(\mathbf{t}_h)] \text{ or “0” } \mathbf{q}_i[N(\mathbf{t}_h)]\}.$$

If $\mathbf{p}_i[N(\mathbf{t}_h)]_{[0,1]} = [N(\mathbf{t}_h)/N]$, then the system’s macro behaviour is *irreversible* and the diffusion accelerates, since the probabilities depend on the synthetic state and this is monotonically non-decreasing⁷.

If we abandon the hypothesis of irreversibility and consider the probabilities associated with the cells in the form $\mathbf{p}_i[N(\mathbf{t}_h)]_{[-1, 1]}$ – so that we admit that a cell could change its state from “0” to “1” as well as from “1” to “0” – then the Combinatory Automaton might show a chaotic macro behaviour in the sequence $X(A, \mathbf{T}) = N(\mathbf{t}_h), h=0, 1, 2, \dots$

To produce *chaotic behaviour* of the automaton, let us assume, for example, that the probabilities are not agent-dependent and take on the following values corresponding to a tent map:

$$\mathbf{p}[N(\mathbf{t}_h)]_{[-1, 1]} = \begin{cases} 2[N(\mathbf{t}_h)/N] & \text{if } 0 < N(\mathbf{t}_h) \leq N/2 \\ 1 - [(2N(\mathbf{t}_h) - N)/N] & \text{if } N/2 < N(\mathbf{t}_h) \leq N \end{cases}$$

If we simulate the micro behaviour by some experiment that generates *random numbers* for each element, we can observe that the combinatory system presents a chaotic macro behaviour, independently of the initial random impulse, $A(\mathbf{t}_0)$, that shapes $\mathbf{a}_i(\mathbf{t}_0)$, as shown in figure 12)⁸.

It is striking to observe how the random dynamics of the combinatory system shown in figure 12[A]-[C] are analogous to the chaotic behaviour shown by the simple quadratic function in figure 12[D] and, in particular, to the effects of path dependence.

In order to simulate accumulation effects, let us assume an irreversible automaton and specify, in the previous point 6) of model [A] the following rule of transition of state:

$$\mathbf{a}_i(\mathbf{t}_{h+1}) = \{\text{“ } \mathbf{a}_i(\mathbf{t}_h) + 1 \text{” } \mathbf{p}_i[\mathbf{M}(A, \mathbf{t}_h)] \text{ or “} \mathbf{a}_i(\mathbf{t}_h)\text{” } \mathbf{q}_i[\mathbf{M}(A, \mathbf{t}_h)]\}$$

In these conditions, the *synthetic state*, $X(\mathbf{t}_h) = \sum_{1 \leq i \leq N} \mathbf{a}_i(\mathbf{t}_h) = N(\mathbf{t}_h)$, continuously increases and the rate of increase depends on the defined probability function.

14 – Conclusions: exploring collectivities through the Combinatory System Theory

⁷ We could also add a *premium* proximity, in order to increase the probabilities of the neighbourhood of the cell whose value has changed from “0” to “1”, which would make the diffusion effect much faster or slower.

⁸ So that the system’s history is irreversible and the system’s future unpredictable, or even chaotic, as the description of regularities is impossible (in the sense of Gell-Mann 1995-96, of Wolfram’s classification scheme (1984, 1994), or of Devaney 1989).

With the aid of unsophisticated *Combinatory Automata* I have tried to demonstrate that even simple collectivities of similar agents developing analogous micro behaviours may show interesting kinds of self-organization and operate following the logic of *synergetics* and *complex systems*.

The *Theory of Combinatory Systems*⁹ searches for the conditions that produce the macro behaviours and proposes models to interpret the collective phenomena. In particular, the theory focuses on the necessity both of recognizing the nature of the global variables that act as global information and of understanding the nature of the macro rules, which specify the recombining factor(s), and of the micro rules, which specify the necessitating factor(s); the joint action of these factors gives rise to and maintains the macro and micro behaviours.

The Theory also considers *reversible* systems (Lustick 2000) that have a cyclical behaviour and, under certain conditions concerning the probability function regarding the transition of state of the agents, a *chaotic* one as well (Gleick 1988, Kellert 1993).

Combinatory systems are *recursively closed systems*; their dynamics are prevalently due to the joint action of "chance" and "necessity"; they might thus also be called "chance-necessity" systems.¹⁰

Other relevant characteristics (I will only mention these) concern the fact that, even though combinatory systems are unorganized and closed systems, they can organize themselves into specialized subsystems and show ramifications (Monod 1971, Maturana and Varela 1987), and can expand their effects on elements belonging to a vaster environment.

Combinatory systems constitute a particular class of complex systems (Gell-Mann 1995-96); but as they follow the simplest schema of adaptation, and because of the similarity of agents and behaviours, the absence of organizational or social links, levels, specializations, multidimensionality and, particularly, direct interactions, cooperation or competition among the agents and their neighbourhood, we could provocatively define these collectivities as a *simplex system*¹¹.

⁹ A general review of the Theory of Combinatory Systems is at the site: www.ea2000.it/cst.

¹⁰ We have used, though with a different meaning, the same terminology used by Monod (1971), who, in his famous *Chance and Necessity*, examined a very powerful combinatory system: that leading to a dynamic evolution in a population due to random mutations produced in the DNA that "by necessity" spread as a result of the invariant reproductive mechanism of cells.

Haken also speaks of *chance* and *necessity* when he proposes constructing models of complex systems. Here Haken considers *chance* as the unpredictable fluctuation from an unstable equilibrium state, and *necessity* as the movement towards a new, more stable state (Haken 1983, Prigogine and Stengers 1984).

Chance will not only set under way the macro behaviour but will also determine the direction, that is the direction of the "winning" fluctuation. Prigogine bases his theory on the emergence of order in complex systems on the consequences of fluctuations (Nicolis and Prigogine 1989, Haken 1983).

A simple way to observe the influence of the random fluctuations in orientating the *direction* of the "macro" dynamics of combinatory systems - even if it is not sufficient to describe the effect of *chance* on the overall dynamics of a combinatory system - is offered by the *Polya Urns* and by the *Ehrenfest Urns*.

¹¹ *Combinatory systems* differ from *complex systems* and, in particular, from *complex adaptive systems* (CAS) and from *Holarchies* in many aspects.

Firstly, because *combinatory systems* do not necessarily present phenomena of adaptation but, generally, some form of self-organization due to the *micro-macro feedback*, that is the adaptation of agents to a synthetic variable produced by the macro behaviour of the system. Adaptation may be a characteristic of some particular class of CS representing populations and not, in general, of collectivities conceived in a broader sense.

A *second* difference is observable also as regards the similarity of the agents: "Here we confront directly the issues, and the questions, that distinguish CAS from other kinds of systems. One of the most obvious of these distinctions is the diversity of the agents that form CAS. Is this diversity the product of similar mechanisms in different CAS? Another distinction is more subtle, though equally pervasive and important. The interactions of agents in cas is governed by anticipations engendered by learning and long-term adaptation." (Holland 1995: 93).

The *third* main difference regards the absence of interactions among the agents; in combinatory systems agents generally interact only with some macro variable and not each other. The *fourth* relevant difference is that the theory of CAS observes the macro effects of the system produced by the agents that follow a schema or change the schema previously followed. Any micro-macro feedback between the micro behaviours and the schema is considered as a relevant characteristic. *Finally*, ignoring the micro-macro feedback implies that CAS theory only focuses its attention on *necessitating* factors and ignores the *recombining* ones.

If the micro behaviours of the agents are determined exclusively by the macro behaviour, the combinatory system is a *pure simplex system*.

If they depend also on an opportune neighborhood as well as, naturally, on the macro behaviour, the combinatory system is characterized by incomplete and limited information.

Finally, if the agents' behaviour depends only on local rules acting on a defined neighborhood, without considering any micro-macro feedback, the system is a complex system and loses the characteristics of a combinatory system and can be simulated by traditional *cellular automata*.¹²

The combinatory systems approach is *neither a macro* approach, since it also refers to local rules by considering micro behaviours, *nor a micro* approach, since it also includes the macro behaviour in the model of the *system*.

It is rather a *micro-macro* approach, precisely in that the *operating rules*, describing the behaviour of the system, must in some way include not only *local rules* but also the *feedback* between the micro and macro behaviours that acts out over many cycles⁷.

Three aspects of this theory make it particularly effective:

1 - it is not limited to describing the macro behaviour of the unit based on general rules or the individual behaviours based only on local rules, but tries to uncover and explain above all the *feedback* between the macro and micro behaviours or their effects;

2 - to understand the phenomena attributable to the action of combinatory systems the theory tries to uncover and make clear the *necessitating factors* (that cause the micro behaviour of each

For a synthesis, see Table 1.

| Table 1 - How do Combinatory Systems differ from Complex Systems? | | |
|--|---|---|
| Complex systems and Holarchies | Complex Adaptive systems | Combinatory Systems |
| Agents are heterogeneous | Diversity of the agents as a constitutive feature | Agents are similar |
| Agents are interconnected and show hierarchy | The Agents present phenomena of adaptation | Agents are not interconnected |
| Micro behaviours are differentiated | Agents are interconnected | Micro behaviours are analogous |
| Agents act following local rules | Agents act following a schema | Agents act following the micro-macro feedback |
| Decisions are prevalently based on the prisoner's dilemma schema | Decisions are based on forecast and expectations | Decisions follow a simple one column pay-off matrix |

¹² For this reason we cannot in general consider the ants, the swarm and, more generally, the cellular automata approaches as examples of combinatory systems, except in the case where the macro behaviour may affect the micro behaviours of the by creating an "aromatic potential field" by spreading *pheromones*). With their micro behaviours the agents spread pheromone across one site (micro information); the increasing concentration of pheromone (global or macro information) increases the probability that each agent will move in the direction of that site. The micro-macro feedback is quite evident (Zollo, Iandoli and De Maio 2001, Deneubourg and Goss 1989). This behaviour is the consequence of *stigmercy* (Grassé 1959).

⁷ We can consider the *micro-macro feedback* approach as a *meso* approach (Rousseau 1985, House/Rousseau/Hunt 1995). "*Formally defined, meso theory and research concerns the simultaneous study of at least two levels of analysis wherein (a) one or more levels concern individual or group behavioral processes or variables, (b) one or more levels concern organizational processes or variables, and (c) the processes by which the levels of analysis are related are articulated in the form of bridging, or linking, propositions.*" (House/Rousseau/Hunt 1995: 73).; "*Organizations affect behavior and behavior affects organizations.*" (House/Rousseau/Hunt 1995: 83). We can observe, however, that these authors do not consider the *micro-macro feedback* as a construct (Morgeson and Hofmann 1999: 259) useful for investigating organizational behaviour.

Combinatory System Theory follows the bottom up approach of Epstein and Axtell (Epstein and Axtell 1996), but unlike that approach ours considers the micro-macro feedback as the origin of self-organization in Combinatory Systems.

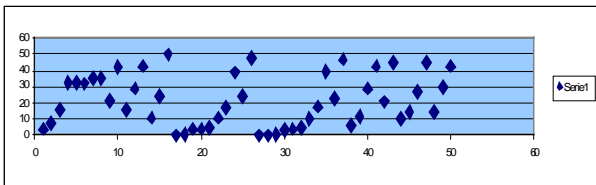
agent in the system) and the *recombining factors* (that produce and maintain the unit's macro behaviour). The theory then concludes that, in the presence of suitable necessitating and recombining factors, "chance" will trigger the dynamic process of the system that "by necessity" is then maintained and influences the individual behaviours;

3 - the procedural explanation offered by the theory not only allows us to understand the operating mechanism that produces the phenomena under examination, but also permits us to determine the most effective forms of control.

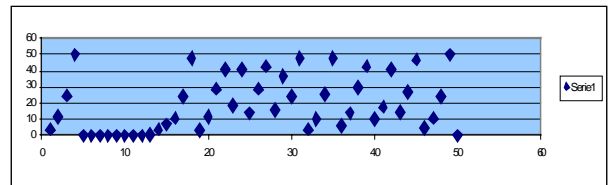
Figure 12 – Reversible probabilistic combinatory system of diffusion with $N = 50$ and showing chaotic macro behaviour. Number of iterations $T = 50$ with neighbouring effects

[A] Changing random numbers

Test [A] 1 - $N(0) = N(1) = 4$

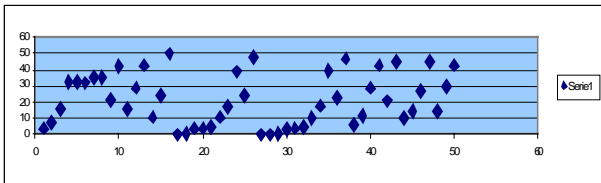


Test [A] 2 - $N(0) = N(1) = 4$ (new random numbers)

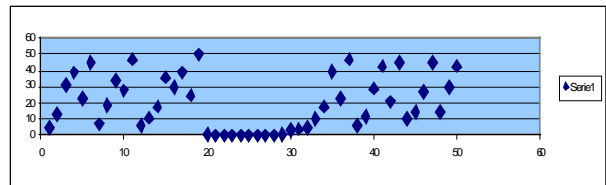


[B] – Changing initial value and keeping the same random numbers

Test [B] 1 - $N(0) = N(1) = 4$

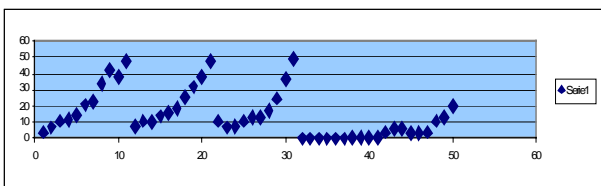


Test [B] 2 - $N(0) = N(1) = 5$

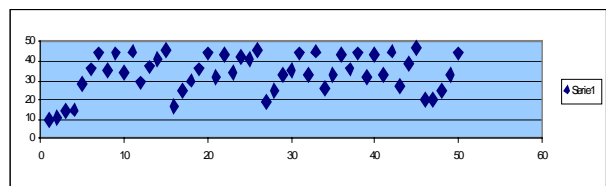


[C] - Probability increases straight line to 1 for $N = 40$ and then decreases to 0 for $N=50$.

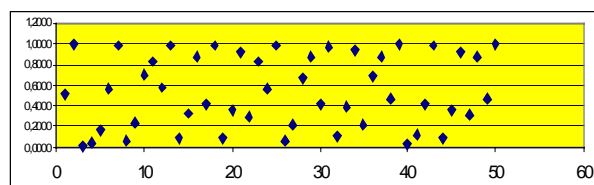
Test [C] 1 - $N(0) = N(1) = 4$



Test [C] 2 - $N(0) = N(1) = 10$ (new random numbers)



[D] - Dynamic system $x = c x (1-x)$ with $c= 3.99$ and $x_0 = 0.85$ for 50 iterations.



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