Nonlinear Dynamics, Psychology, and Life Sciences, Vol. 9, No. 2, April, 2005.

Observing Collectivities as Simplex Systems: The Combinatory Systems Approach

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Abstract: *My paper aims to present a particular class of complex systems made up of collectivities of non- interconnected similar agents, acting on the basis of global information which they directly produce and update by the combination of their analogous micro behaviours, and that directs the subsequent micro behaviours as a result of a micro-macro feedback that produces self-organization in the agents' micro behaviours. Due to their operative logic these systems can in general be called Combinatory Systems. I have provocatively defined the simplest class of them as Simplex Systems, since the similarity of the agents and the micro behaviours, the absence of direct interactions among the agents, and the simplicity of their structure and operative logic make these collectivities a particular simplified class of complex systems, as usually conceived. If we accept the traditional definition of self-organization as the macro behaviour of a collectivity of agents in which the micro behaviours appear to be directed, or organized, by an Invisible Hand, or Supreme Authority, in order to produce the emerging phenomenon represented by the formation of ordered structures, or recognizable patterns, then it is easy to recognize the synergetic effect of a micro-macro feedback action (or circular causality) acting between the agents' micro behaviours and a global system's macro behaviour or effect.*

Key Words: agent-based models, cellular automata, path dependence, combinatory systems, self-organizing processes, chaos, social dynamics

COMBINATORY SYSTEMS THEORY

Strange but Simple Phenomena

We observe many phenomena and processes that, because of the simplicity of their effects, cannot be easily represented and explained either by means of traditional system theory – that views systems as

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black boxes that operate according to an [input-Δstate-output] logic - or by the logic of complex systems, whose dynamics is produced by local rules directing the agents' behaviours (see below).

How do we explain the birth of paths in fields? Why do industrial innovations quickly spread? What is the force behind the continual improvement in the quality of products? Why are some park benches or walls covered by graffiti while others nearby are spotless? How does a feud develop? Why, in Pavia, in the span of a few decades, have over 100 towers been built, all alike, without any apparent function, if not a symbolic one? Why do dangerous wheel tracks form in certain sections of highway while in others the asphalt, which is of the same quality, appears to resist the stress of traffic? How many times have we witnessed the breaking out of applause after an initial uncertainty; and why at other times has applause not arisen when the same conditions exist? How do we explain urban and industrial settlements in circumscribed areas? What mechanism can we use to explain the maintenance of languages and dialects in limited areas? Why do mounds of garbage spontaneously form in certain areas?

Why do individuals chase records? Why does a background murmur arise in crowded rooms, which causes everyone present to talk in a louder voice? Why are speed limits and no standing rules invariably ignored despite rigid controls? How does spontaneous order generally arise in a ballroom when a waltz is played? And what about the grouping of flocks, swarms, herds and other collectivities of animals? Or the mechanism followed by the Can-Can dancers at the Moulin Rouge to remain aligned? How did the Hoplites create a Macedonian phalanx and maintain its order? Why in England does everyone line up in an orderly queue while waiting their turn, while in other "Latin" countries people wait their turn in a disorderly fashion? The same logic of the stadium wave applies, which transforms a disorderly group of spectators into a wave that runs several times around the stadium.

In order to develop a system thinking (von Bertalanffy, 1968) capable of explaining the operative mechanisms that regulate these phenomena I propose the *Combinatory System Theory*, based on the idea of a *Combinatory System*, defined as a non-organized system of relatively similar agents, producing relatively analogous micro behaviours, that lead to observable (or definable) micro effects; *combined together* the micro behaviours and effects produce a macro behaviour and a macro effect that, in turn, conditions the subsequent micro behaviours of the agents, according to a *micro-macro feedback* acting over a period.

They Appear Guided by an Invisible Hand

In effect it is easy to recognize that in all the examples indicated above we observe an apparently strange process: on the one hand, the macro behaviour - and the macro effect that derives from this – seems produced by a *combination* of micro behaviours of a plurality of agents but, on the other, it seems to guide the micro behaviours of the agents, as if an Invisible Hand, or Supreme Authority, or Benevolent Deity forced the individuals to produce the emerging phenomenon represented by the formation of ordered structures, of recognizable patterns (Foster & Metcalfe, 2001; Pelikan, 2001).

There is nothing strange or metaphysical here: The *Invisible Hand* is nothing other than the *synergetic effect* of a *micro-macro feedback* action (or *circular causality*) acting between the agents' *micro behaviours* and a global system's *macro behaviour* or *effect* as shown in Fig. 1. In this sense all the phenomena mentioned in the previous section, and others that will be analyzed in the following ones, are clear examples of *self-organization* or *spontaneous order* (Ashford, 1999; Kauffman, 1993; Sugden, 1989; Swenson, 2000) that can give rise to orderly or chaotic behaviour, to orderly or disorderly effects.

Adam Smith's invisible hand naturally comes to mind. Adam Smith used the term "invisible hand" only once in his Wealth of Nations: *"...[By] directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention. Nor is it always the worse for the society that it was not part of it."* (Smith, 1776, p. 456)

The invisible hand was also mentioned by Haken, the founder of Synergetics, when he tries to explain the synergetic effect in complex systems: "*We find that the various parts are arranged as if guided by an invisible hand and, on the other hand, it is the individual systems themselves that in turn create this invisible hand by means of the coordinated effect. We shall call this invisible hand that gives order to everything the 'organizer'*" (Haken, 1983, p. 17).

The *micro-macro* feedback may be thought of as an *internal dynamic director* or, better yet, as an *internal dynamic organizer* which produces and uses *global information* as an *order parameter* and, following the *slaving principle*, directs or organizes the individual behaviours and produces the self-organization of the system and hence the collective phenomena (von Foerster, 1960; Haken 1977; Kauffman, 1993; Martelli, 1999; Prigogine, 1984).

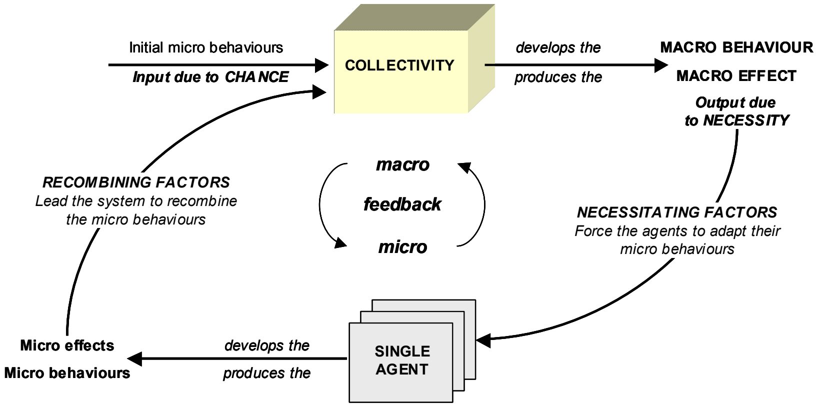


Fig. 1. Chance and necessity.

Analogous Explanation

To construct a general model of *Combinatory System*, let us try to recognize the common features of the phenomena mentioned in the previous sections. *Above all*, we immediately observe that these derive from, or are caused by, a *collectivity* or a plurality of non-similar agents (thus non-organized, in the opposite sense with respect to Maturana & Varela's, (1980) concept of organization) which produce an analogous micro behaviour over time – or similar micro effects – but, considered together, are capable of developing a macro behaviour – and/or macro effects – which is attributed to the collectivity as a whole.

The graffiti is left by a multiplicity of people who, day after day, write or carve names, sayings, initials and messages; and a garbage pile is also the result of numerous acts involving people leaving garbage behind; the Macedonian phalanx is made up of a large number of Hoplites; highway carriageways are the result of the passing of countless numbers of trucks; records are achieved by the action of thousands of athletes who, in different places and at different times, compete in order to surpass the existing record; applause does not break out, and the murmur in a room does not arise, if there are not enough people present.

The *macro behaviour* of the system is the collective action we attribute to the collectivity (walking down the same path or driving along the same stretch of highway, the wave at the stadium, the circular dance of the ballroom dancers, sitting on the same bench and carving initials in it, and so on). Its observable effects (path, carriageways, the wave circling the stadium, rotating motion, graffiti, the catching on of a fashion trend, etc.) are defined as the *macro effects* of the system. But how does a murmur arise in a room if the individuals do not speak to each other? How does applause break out if the individual spectators don't applaud? And how does a fashion spread if those belonging to the collectivity do not develop imitating behaviour? Though the mass of graffiti comes from a plurality of people, it is nevertheless formed by individual graffiti writings.

Thus a *second observation*: the macro behaviour and correlated macro effects result from the collective action, in that each of its elements produces a distinct *micro behaviour*, which can produce a particular *micro effect.* A pair of individuals (elements) speak (micro behaviour) at a given voice level (micro effect). The crowded room (collectivity) of talking people (macro behaviour) create a background noise (macro effect). Many individuals (collectivity) cross a field (macro behaviour); each individual (agent), while crossing (micro behaviour), steps on the grass (micro effect) and a path is thus formed (macro effect).

In order to fully explain collective phenomena we must, however, take a *third step*: Determine the mechanism that connects their micro and macro behaviour or their micro and macro effects. To easily understand this mechanism, let us again consider the phenomenon of a murmur arising in a crowded room: the macro effect (murmur) from the macro behaviour (collective chatter) depends on the micro effect (voice level) produced by the micro behaviour (speaking in pairs); reciprocally, the macro behaviour (exchange of information among those present) and the macro effect (murmur) influence the micro behaviour (attempts at communicating) and the micro effects (voice level).

This mutual dependence between the micro and macro behaviour (or their effects) can be defined as *micro-macro feedback*, as represented in the model shown in Fig. 1. Let us consider the Combinatory System observed during the transmission process of a language within a population. Each parent transmits the mother tongue (micro behaviour) to its children and the children learn it (micro effect). The children make up the population (system) that speaks the mother tongue (macro behaviour). The existence of a mother tongue (macro effect) forces the families to teach that language to their children. The feedback is clear.

Recognizing the action of a *micro-macro feedback* loop is a necessary and sufficient condition to define a collective phenomenon as deriving from a Combinatory System. The existence of *micro-macro feedback* thus leads to an essential consequence: The macro behaviour of the system cannot be considered a mere sum of the micro behaviour of its elements (or of their effects); the *micro-macro feedback* causes "emerging" types of macro behaviour (or effects) which are attributable to the unit.

"Chance" and "Necessity"

Although the three characteristics presented in the above section are fundamental, these features are not sufficient to understand the formation of collective phenomena. We must recognize another essential element: the joint action of "chance" and "necessity". Most collective phenomena begin when "by chance" a *minimum density* (or *activation level*, specific to each phenomenon) is reached, and it ends when a *maximum density* is reached (that is, a given *saturation level*).

Once the minimum density is reached, the *micro-macro feedback* guarantees that the macro behaviour "by necessity" initiates and grows, feeding on the subsequent micro behaviours and, at the same time, conditioning them. The activity of the Combinatory System is thus produced by the joint action of "chance" and "necessity"; they can therefore also be called "chance-necessity" systems.

Let us now consider the Combinatory System that explains how graffiti comes to appear on a wall or a bench. It is absolutely clear that the initial graffiti is placed there by chance by someone who wishes to leave behind a trace of himself; if it is removed the system does not start up; if it is left there, and if by chance others leave behind new graffiti, the system by necessity leads to the formation of a new mass of graffiti. I have used, though with a different meaning, the same terminology used by Monod (1971), who, in his famous *Chance and Necessity*, examined a very powerful Combinatory System: that leading to a dynamic evolution in a population due to random mutations produced in the DNA that "by necessity" spread as a result of the invariant reproductive mechanism of cells. Haken also speaks of *chance* and *necessity* when he proposes constructing models of complex systems. Here Haken considers *chance* as the unpredictable fluctuation from an unstable equilibrium state, and *necessity* as the movement towards a new, more stable state (Haken, 1983; Prigogine & Stengers, 1984). Chance will not only set under way the macro behaviour but will also determine the direction, that is the direction of the "winning" fluctuation. Prigogine bases his theory on the emergence of order in complex systems on the consequences of fluctuations (Haken, 1983; Nicolis & Prigogine, 1989).

Necessitating and Recombining Factors

In order to observe concrete collectivities, recognizing the action of *chance* acting together with the *micro-macro feedback* is necessary but not yet sufficient to understand the macro behaviour and the phenomena we wish to explain. The knowledge of the factors which produce and maintain the micro-macro feedback is thus indispensable. We define them as *necessitating* and *recombining* factors. *Necessitating* factors are the factors that *force* the agents to adapt their micro behaviour to the system's macro behaviour. Often these necessitating factors come from obligation, convenience, utility, or the desires of the individual elements; at other times they act without the individuals being aware of it. The existence of one or more *necessitating* factors is indispensable but not sufficient. A set of *recombining* factors that lead the system to recombine the micro behaviours (or the micro effects) in order to produce and maintain the macro behaviour (or the macro effect) are thus necessary because, if the collectivity in some way is not able to "join together" their micro behaviours (or their effects), then the *micro-macro feedback* cannot occur. Considering also the necessitating and recombining factors, we modify the model in Fig. 1 to look like that in Fig. 2.

Several necessitating factors are strong at times. Think of the power of the survival instinct that, in the presence of a predatory fish, creates in a school of small fish the need to hide within the group; if, by chance, some fish in flight come together and form an initial grouping, the instinct to hide (necessity) attracts other fish, and very soon we have the formation of a compact mass; this continues as long as there is still the threat from the predatory fish, which represents the recombining factor; if the danger subsides the mass of fish soon disappears.

Consider, on the other hand, the much higher degree of need a family has in passing on its language (conscious need) and its dialectal inflection (unconscious need) to its children. The necessitating factor can even be natural and act in an unconscious way. In the system characterized by the spread of an epidemic the necessitating action does not depend on the desire of the carrier of the pathogenic agent to infect other individuals, as much as on the operative aggression mechanism of the virus. The recombining effect usually derives from the environment in which the collectivity operates; at other times from conventions or constraints which are external to the system.

Consider how the recombining effect from the passing of trucks to form carriageways on the highway differs according to whether or not the stretch of road is straight or winding. In a large environment it is much simpler to scatter one's garbage than to accumulate it.

Local and Global Information

To complete this brief presentation of the logic of Combinatory Systems we can consider the phenomenon of *synchronization* that we can observe in most Combinatory Systems: here the agents seem to simultaneously produce or coordinate their micro behaviours.

A certain number of persons attend an event. Suddenly someone – by *chance* or *directed* by someone – claps (micro behaviour), thereby producing a typical sound (micro effect). If the number of those that begin to clap does not reach the *minimum activation number*, then the applause does not begin. But if the initial clapping does not *die down*, others will join in and there is thundering applause. The micro behaviours translate into a macro behaviour (everyone applauding), of which the applause, understood as a typical sound, represents the macro effect and the global information according to which the subsequent micro behaviours are synchronized.

Another case of synchronization is shown by certain insects (Deneubourg & Goss, 1989), typically ants, which create an "aromatic potential field" by spreading *pheromones* or other permanent messages (Zollo, Iandoli & De Maio, 2001), and are thus able to trigger a chain reaction (Grassé, 1959). The answer must be found in the mechanism according to which the individual elements in the system produce their micro behaviours.

The behaviour of the collectivity can be defined as *local* – or based on *limited information* – or global – or based on *complete information* – depending on whether or not the macro behaviour derives from *local* information possessed by the agents (a person acquires a good because he observes that at least *N* friends have bought it; an elephant in a herd runs to the left because the elephants on its right push it in this direction) or from *global* information (over time and/or space) possessed by all the agents (all people raise their voice because the noise increases, all the animals flee because they see the fire advancing). *Global information* may derive from outside (external director, starting traffic lights or the starting gun in races, trumpeting by the leader of the herd, and so on) or may be the macro behaviour or the macro effect *self-produced* by the collectivity as a whole. We can observe that Combinatory Systems can be also conceived as collectivities of agents whose micro behaviours are based on a global information (the cloud of graffiti, the pile of garbage, the applause, the carriageway, the feud, the leader agent, the mean diffusion, and so on) that they self-produce and self-update, following a *micro-macro feedback*.

Control of the Phenomena

We have seen that to understand the operative mechanisms of a Combinatory System we need to identify the necessitating and recombining factors that support the *micro-macro feedback*.

With the formation of the path, for example, the necessitating factor could be represented by the desire not to dirty one's shoes. So seeing the freshly trodden grass (global information) the passer-by follows the trail. The recombining factor is the presence of grass that remains trodden and the frequency of the comings and goings of the passers-by. If the ground were rocky and the frequency of the passings low, no path would form.

This example brings out a *final* important consideration: if the path were considered to be useful, in that it indicated the right direction and an easy way to cross the field, red signs or even kerbstones could be placed there, or the public authorities could intervene to enlarge the trail and pave it, so that the Combinatory System would be *institutionalized*. Let us assume instead that the path crosses the field of a farmer who sees in that macro effect a threat to his crops. Under this assumption it is easy to imagine the various actions to defend his property, the most drastic of which might be to enclose the field with walls. This would cause the Combinatory System to cease to operate.

That is why in the presence of apparently similar conditions (collectivity, individuals, necessitating and recombining factors) the phenomena can present rather diverse developments. In some circumstances the environment "strengthens" the system (or its macro behaviour or derived macro effects) since they are held to be useful; at other times the system instead undergoes an action aimed at weakening the macro behaviour or the macro effects, and eventually disappears.

Thanks to timely strengthening and weakening actions it is possible to control the Combinatory System; that is, to make sure the macro behaviour is the desired one. The strengthening and weakening actions can act on both the macro behaviour (or on the macro effect) in order to produce a control at the macro level, or external control, as well as on the micro behaviour (and its micro effects). In this case we speak of a micro-level or internal control. Figure 1 can now be completed as shown in Fig. 2.

Assume that in order to combat the spread of drugs the government undertakes an anti-drug policy or blocks the "traffic" at the point of origin, thereby eliminating imports. This certainly represents a weakening action by which there arises a form of external control on the macro behaviour. Let us instead suppose that individual families develop a form of anti-drug campaign that creates in young people the desire to avoid taking drugs. In this case the system would modify its own macro behaviour through the control exercised on the micro behaviours by the elements which form its collectivity.

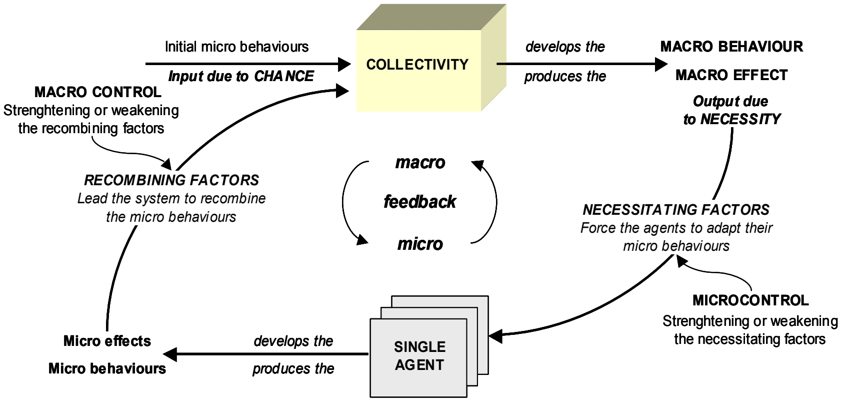


Fig. 2. Macro and micro control of a Combinatory System.

SIMPLEX SYSTEMS
The Main Definition

The most interesting Combinatory Systems are those I have provocatively defined as *Simplex Systems*, since the similarity of the agents and the micro behaviours, the absence of direct interactions among the agents, and the simplicity of their structure and operative logic, make these collectivities a particular *simplified* class of complex systems as usually conceived.

I define as a (social) simplex system any collectivity acting on the basis of self-produced *global information* and showing the following functioning rules: (a) All agents are similar in the sense they show a relatively similar nature, structure or significance; (b) the agents are not necessarily interconnected by evident interactions, or by network, web or tree structures; (c) all the agents are characterized by the same individual variable (or set of variables) of some kind whose values – at any time t_h – represent the individual micro states whose dynamics – over a period T – may be defined as the micro behaviours of the agents, which may lead to analogous *micro effects* of some kind; (d) the collectivity, as a whole, is characterized by a *macro (global) variable* whose values – at any time t_h – derive from the *combination* appropriately specified (sum, product, average, min, max, etc.) of the agent's states and represent the *system's macro states* whose dynamics – over a period T – may be defined as a *macro behaviour* of the collectivity; (e) the *macro behaviour* in turn may lead to a *macro effect* of some kind that constitutes the output of the system and may be conceived – or interpreted – as *global information* for the agents; (f) each agent – at time t_{h+1} – through the *global information*, can perceive and evaluate – in a simple pay-off table – positive or negative gaps (advantages or disadvantages) between his individual state and the state of the collectivity; (g) each agent then makes individual micro decisions, by processes of rational choice or by imitation or social learning (Conte & Paolucci, 2001) in order to increase (if positive) or reduce (if negative) the perceived gaps, thereby changing its *micro behaviour*; (h) but, these decisions recursively change the value assumed by the *macro behaviour*, that is the global information, and this modifies the perceived positive or negative gaps, driving the agents to adapt their behaviour by new decisions.

Self-organization and Synchronization in Simplex Systems

The *operative logic of Simplex Systems* is as basic as their structure as shown in Fig. 3: (a) O*n the one hand* the agents, *consciously or unconsciously, act (exclusively or prevalently) on the basis of* global information *which they directly produce and update as the consequence of their micro behaviours;* they thus seem self-organized to produce the macro behaviour of the system which, for an observer, may be conceived as an *emergent phenomenon;(b) on the other hand*, the macro behaviour updates the global information and determines, conditions, directs, or drives the subsequent micro behaviours in a typical *micro-macro feedback;* this, for an observer, may be conceived as a *self-organization effect*; (c) *the micro-macro feedback* operates between the limits of the *minimum activation number* and the maximum saturation number of the agents presenting the state that maintains the *micro-macro feedback*; this guarantees over time both the production of the emergent phenomenon and the maintenance of the self-organization effect; (d) since by definition the agents are similar and have similar behaviour, it follows that we can assume that the same information produces similar decisions regarding the change in state of the agents, who thus appear to conform or even *synchronize* their micro behaviours, as we can observe in the process of applause.

Models of Simplex Systems

To understand collective phenomena we must try to build a model that represents, in a clear and simple way, the operative mechanisms of the Combinatory or Simplex Systems that these phenomena produce. Combinatory systems can be represented by different models of increasing complexity.

The simplest models are the *descriptive ones* that indicate in words the fundamental elements necessary for understanding the operative logic of systems that produce observable collective phenomena. More powerful are the *heuristic models* that try to simulate the system's dynamics by stating - or constructing *ad hoc* – a set of rules specifying: (a) The *micro*, or *necessitating*, rules producing the micro behaviours of agents; (b) the *macro*, or *recombining* rules that produce the *system's macro behaviour*; (c) the *micro-macro feedback* that allows the system to produce the observed phenomena.

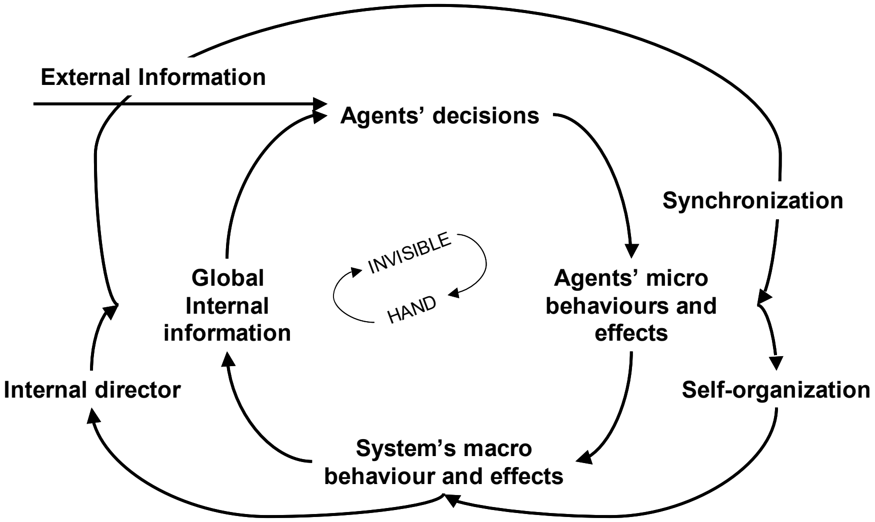


Fig. 3. Self-organization and synchronization in Simplex Systems.

For example, the *voice-noise* simplex system can be represented by the following rules: MICRO RULE = NECESSITATING FACTOR: if you have to talk and you hear a background murmur or noise, raise your voice level several decibels above the background noise; when the maximum bearing is reached, keep quiet. MACRO RULE = RECOMBINING FACTOR: the collectivity makes interpersonal communication necessary or favors it; the environment preserves the noise and also takes account of the noise factor arising from causal factors (bells ringing, shouts from outside the system's environment, etc.). MICRO-MACRO FEEDBACK: the murmur depends on the volume of the voices of the individual speakers; but if the background noise increases, then the speakers will also raise their voices, which in turn will cause an increase in the murmur. When the maximum bearing level is reached the murmur ceases. If «by chance» someone should start speaking again, even to say «What silence!», then «by necessity» the background noise will reappear and the Combinatory Systems will start to operate.

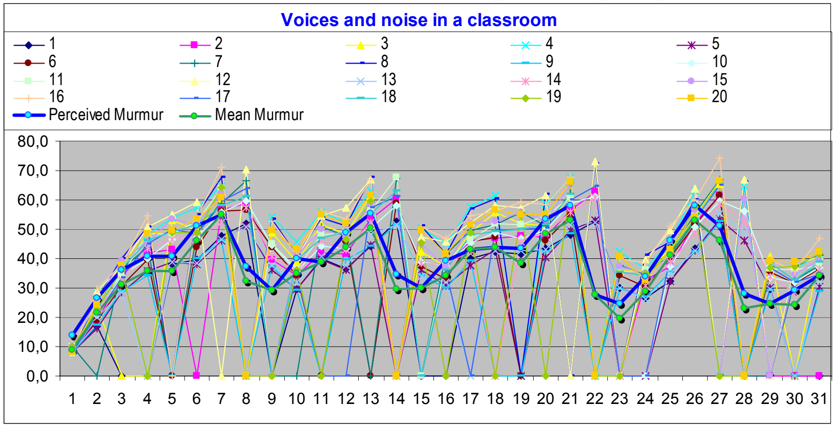
Finally, we can build a *combinatory automaton* that specifies the *mathematical* and statistical simulation model that represents the behaviour of the Combinatory Systems (Mella, 2003). The rules established in the heuristic model of the voice-noise simplex system can be easily translated into a combinatory automaton in which the murmur – $M(\Lambda, t_h)$ – is the output of the crowded room considered as a collectivity and is produced by the combination of the voice levels – $v_i(t_{h+1})$ – of the individual speakers who, in order to make themselves heard, must raise their voices some decibels – $v_{i(min)}$ – above the murmur. But recursively this increases the murmur, in a typical feedback between micro and macro behaviour. We can represent this phenomenon through the *stochastic combinatory automaton* (see Mella, 2001):

$$\begin{cases} \Lambda(\mathrm{t}_0) = \mathrm{v}_\mathrm{i}(\mathrm{t}_0) \leftarrow \text{“CHANCE”} & 1 \le i \le N \\ \mathbf{C}_{1\le \mathrm{i} \le \mathrm{N}}\,[\mathrm{v}_\mathrm{i}(\mathrm{t}_\mathrm{h})] = (1/\mathrm{N})\,\Sigma_{1\le \mathrm{i} \le \mathrm{N}}\,\mathrm{v}_\mathrm{i}(\mathrm{t}_\mathrm{h}) & \\ \mathrm{M}(\Lambda, \mathrm{t}_\mathrm{h}) = \{\, \mathrm{k}\,[(1/\mathrm{N})\,\Sigma_{1\le \mathrm{i} \le \mathrm{N}}\,\mathrm{v}_\mathrm{i}(\mathrm{t}_\mathrm{h})] + \mathrm{Q}\,\mathrm{r}(\mathrm{t}_\mathrm{h})_{[0,1]} \,\}\,(1 - \mathrm{a}) & h = 0, 1, 2, \ldots \\ \mathrm{v}_\mathrm{i}(\mathrm{t}_{\mathrm{h}+1}) = \{\, [\mathrm{w}_\mathrm{i}\,\mathrm{M}(\Lambda, \mathrm{t}_\mathrm{h}) + \mathrm{v}_{\mathrm{i(min)}}] + \mathrm{v}_{\mathrm{i\ (rnd)}}\,\mathrm{l}_\mathrm{i}(\mathrm{t}_\mathrm{h})_{[0,1]} \,\}\,\mathrm{s}_{\mathrm{i}\ [0,1]}\,\mathrm{b}_{\mathrm{i\ (bol)}}(\mathrm{t}_\mathrm{h}); & 1 \le i \le N \end{cases}$$

The crowded room recombines the voice levels into a simple mean $[(1/N)\ \Sigma_{1\le i\le N}\ v_i(t_h)]$, but the noise level also depends on several factors – the nature of the speakers, the necessity of speaking, the structure of the room that, recombining the voices, can maintain or reduce the murmur – which specify a set of appropriate parameters for the macro and micro functions F and f.

In *particular*, the *necessity to speak* is represented by a probability, $s_{i\ [0,1]}$, that may or may not depend on time and on the number of talking people. If we introduce *tolerance*, $b_{i\ (bol)}\ (t_h)$ into the model, that is the maximum level of bearing, then the system may show a cyclical behaviour. The simulation model of Fig. 4 shows this phenomenon; it describes a linear *stochastic medial automaton* of 20 (non-ordered) speakers observed for 30 iterations. The voice levels (simple lines) are the variables associated with the speaking agents. The noise (bold line) may be viewed as the output of the combinatory automaton constituting the collectivity considered as a whole.

Test 1 – External noise Q = 5 dec. Mean probability to speak = 87%



Test 2 – *External noise Q = 10 dec. Mean probability to speak = 90%*

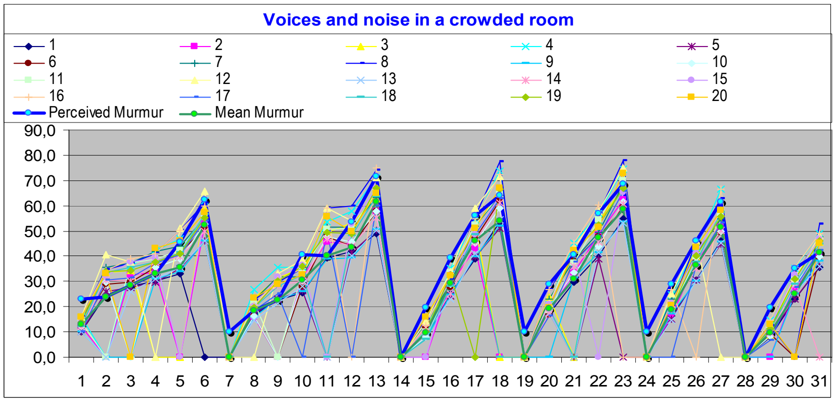


Fig. 4. Model of Murmur and Noise system with 20 agents and differentiated probabilities for each agent.

A Bit of Order, Types of Simplex or Combinatory Systems

If we classify Combinatory Systems according to their macro behaviour (or their macro effect) we can, despite the variety of phenomena produced, determine five fundamental types of Simplex or Combinatory Systems.

Systems of "Accumulation"

We can define as "accumulation" systems those Combinatory Systems whose macro behaviour leads to a macro effect that can be perceived as an accumulation of objects, types of behaviour, or effects of some kind. The following heuristic model can describe these systems: NECESSITATING RULE: if you have to accumulate some object with others similar in nature (micro behaviour), look for already-made accumulations, since this gives you an advantage or reduces some disadvantage (necessitating factor); RECOMBINING RULE: the environment preserves the accumulated objects or is not able to eliminate them, and maintains the advantages of the accumulation; everyone accumulates (macro behaviour) and an accumulation of some kind is created (macro effect); MICRO-MACRO FEEDBACK: the larger the accumulation (macro effect), the more incentive (facility, probability) there is to accumulate (micro behaviours) objects (micro effects); the collective accumulation (macro behaviour) leads to the maintenance or the increase of the accumulation.

This logic applies to quite a diverse range of phenomena, among which the formation of urban or industrial settlements of the same kind and of industrial districts, the accumulation of garbage, graffiti, writings on walls; but it can also be applied to phenomena such as the breaking out of applause, the formation of lines in fashion shows, the grouping of stores of the same type along the same street.

Systems of "Diffusion"

This term can be used for all systems having as a macro effect the diffusion of a feature, a peculiarity, or a "state" from a limited number to a high number of elements of the system. Heuristic model: NECESSITATING RULE: if you see that an "object" is diffused, then it is "useful" for you to possess it or harmful not to possess it (necessitating factor), and you must try to acquire it; RECOMBINING RULE: the environment or the collectivity preserves the diffused objects and maintains the utility of possessing the object; the higher the utility or need to acquire the object for the individuals, the more the object will spread throughout the collectivity; MICRO-MACRO FEEDBACK: a greater diffusion (macro effect) implies a greater desire to acquire the object (micro effect); the single acquisition (micro behaviour) widens the collective diffusion (macro behaviour).

Systems of diffusion explain quite a diverse range of phenomena: From the spread of a fashion to that of epidemics and drugs; from the appearance of monuments of the same type in the same place (the towers of Pavia, for example) to the spread and maintenance of a mother tongue, or of customs.

Systems of “Pursuit"

We give this name to the Combinatory Systems that produce a behaviour consisting in a gradual shift of the system toward an objective, just as if the system, as a single entity, were pursuing a goal or trying to move toward ever more "advanced" states. Heuristic model: NECESSITATING RULE: if there is an objective, try to achieve it; if there is a limit, try to exceed it; if another individual overtakes you (negative gap), regain the lost ground; if you're even with someone, try to go ahead; if you're in the lead, try to maintain or increase your advantage (positive gap); RECOMBINING RULE: the collectivity recognizes the validity of the object and views limits in a negative way; the more individuals try to exceed the limit, the greater the chance of exceeding it, with a consequent advantage for those who succeed in doing so. This provides the incentive for the pursuit; MICRO-MACRO FEEDBACK: if everyone tries to go beyond the limit (macro behaviour), then this is raised (macro effect), thereby eliminating the advantage for those who have already reached it (micro effect); this forces the individuals to exceed the limit (micro behaviour).

This model can represent quite a different array of Combinatory Systems: from the pursuit of records of all kinds to the formation of a buzzing in crowded locales; from the start of feuds and tribal wars in all ages to the overcoming of various types of limits.

Systems of “Order"

We thus define Combinatory Systems which produce a macro behaviour or a macro effect capable of interpretation as the attainment and maintenance of an arrangement, an ordered disposition, among the elements that form the system. Heuristic model: NECESSITATING RULE: there are advantages in maintaining a particular order and disadvantages in breaking it; if you want to gain the advantages or avoid the disadvantages, try to adjust your behaviour so that you maintain or achieve the order that is indicated by the rules that establish it; RECOMBINING RULE: the more the particular order is maintained, the greater the advantages from adjusting one's behaviour to maintain it and the disadvantages from breaking it; MICRO-MACRO FEEDBACK: the order (macro effect) creates the convenience for individuals to maintain the arrangement and respect the rules (micro behaviours); everyone maintains a coordinated behaviour (macro behaviour).

The systems of order can be used to interpret a large number of phenomena: from the spontaneous formation of ordered dynamics (for an observer) in crowded places (dance halls, pools, city streets, etc.) to that of groups that proceed in a united manner (herds in flight, flocks of birds, crowds, etc.); from the creation of paths in fields, of wheel-ruts in paved roads, of successions of holes in unpaved roads, to the ordered, and often artificial, arrangement of individuals (stadium wave, Can-Can dancers, Macedonian phalanx).

Systems of “Improvement” and "Progress"

A very special and important combinatory system is the one I have named the *Improvement and Progress Combinatory System*, since its particular effect is to produce progress, understood as an improvement in the overall state of a collectivity that is attained through individual improvement.

These systems can be classified among those belonging to the classes mentioned above; in particular they are systems of pursuit that produce accumulation or diffusion; I shall describe them as an independent class only because of their particular relevance in social collectivities. Individual improvements raise the parameter that measures collective progress; this constitutes the global information that leads to the perception of positive and negative gaps that push the individuals to improve in order to increase the gaps (if positive) or eliminate them (if negative) (Fig. 5).

The system must be able to notice the individual improvement and to adjust the progress parameter to the average (or, more generally, to the combination) of the individual improvement measures. Heuristic model: NECESSITATING RULE: if you perceive that the level of your improvement parameter is below the level of the system's progress parameter – that is, that there is a negative gap between your state and that of the others – try to improve in order to reduce the gap and, if possible, try to attain a positive gap; if you perceive there is a positive gap, do nothing or try to improve further in order to increase the favourable gap; RECOMBINING RULE: the system must be able to notice the individual improvement and adjust the progress parameter to the average (or, more generally, to the combination) of the individual improvement measures; MICRO-MACRO FEEDBACK: individual improvement (micro effect) raises the parameter that measures collective progress (macro effect); this leads to the formation of positive and negative gaps that push the individuals to improve in order to increase the gaps (if positive) or eliminate them (if negative).

Among those phenomena that can be explained using the systems of improvement and progress are the growth of productivity in firms, the continuous improvement in the quality of products, progress in the sciences and in technology, and the evolution of all types of species.

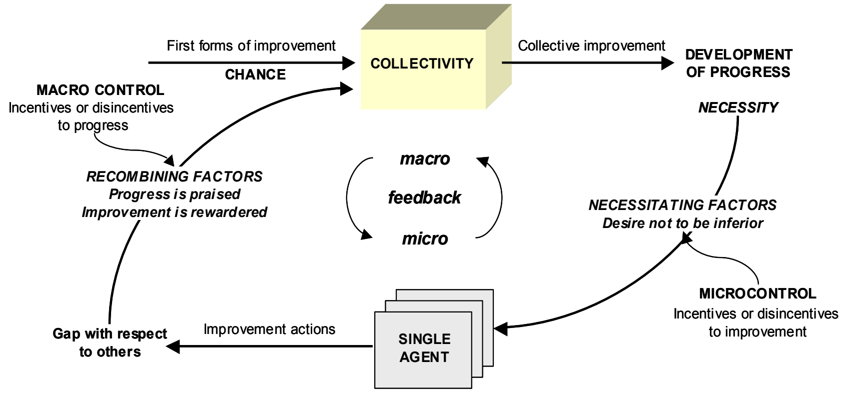


Fig. 5. Model of system of improvement and progress.

Irreversible and Reversible Simplex Systems: Path Dependence and Chaos

In *social* Simplex (or Combinatory) Systems, whose structure is composed of *cognitive agents*, that is of elements that decide to change their state on the basis of global information, probabilities play an essential role for understanding and modeling the systems. In *probabilistic* Simplex (or Combinatory) Systems the micro behaviour depends on a *probability of transition of state*, and is carried out in a *period of transition* of state. Both *probabilities* and *periods* of transition of state nevertheless depend on the state of the system, so that, in turn, the *micro behaviours* are conditioned by the *macro behaviour* of the entire system.

The *probability of transition* expresses the influx of *necessitating factors* and offers numerical information on the likelihood of a given micro behaviour and a given micro effect which can potentially be carried out and obtained by each agent. Due to the existence of the *micro-macro feedback*, even if the state of the system derives from the state of its elements, this nevertheless influences the micro behaviours and the states of the elements in the base according to the probability of transition for each one; a probability that depends, in turn, on the state of the system.

We must therefore take account of this feedback, for example by writing that: (a) The state of each element depends on the probability that characterizes it; but this probability is in turn a function of the state of the system; (b) the length of the period of transition of state of each element that is modified is also a function of the state of the system. The Combinatory Systems that are most interesting and easiest to represent are the *irreversible* ones, where both the micro and macro behaviour produce permanent effects (residential or industrial settlements, the maintenance of the language, the spread of epidemics). *Irreversible systems* explain almost all cases of path dependence (Fuchs & Haken, 1989).

With regard to Simplex Systems, *path dependence* (Arthur, 1988, 1994; Liebowitz & Margolis, 1998) is proof of the action of the *micro-macro feedback*, in the sense that the dynamic of a social system – its macro behaviour or its macro effect – can be thought of as depending on initial chance (dependence from initial conditions) and on the *necessitating* and the *recombining* rules directing the micro behaviours of the agents. Thus, the individual choices of the agents lead to micro behaviour that derives from the past history, that is from the macro behaviour (history dependence).

Combinatory System Theory also considers *reversible systems* (Lustick, 2000), that is, systems whose elements may again show a state that occurred in the past, so that they may present a *cyclical* behaviour and, under certain conditions concerning the probability function regarding the transition of state of the elements, a chaotic one as well (Gleick, 1988; Kellert 1993).

Chaotic Behaviour in Reversible Probabilistic Systems

As an example, consider a non-ordered system where every Agent is a Bernoulli random variable that, at any $t \in T$, shows only a repertoire of two states: {“1” $p_i[N(t_h)]$ or “0” $q_i[N(t_h)]$} according to the *probabilities* of transition from state “0” to state “1”, $p_i[N(t_h)]$, as a function of the number $N(t_h)$ of agents assuming state “1”.

We observe that there is a *feedback between the micro and macro behaviour*, in the sense that the state of each agent depends on $p_i[N(t_h)]$, which in turn depends on the state of the system, $N(t_h)$, which defines the macro behaviour. Let us simply assume that the probability of transition of states takes on the following values:

$$p[N(t_h)] = \begin{cases} 2[N(t_h)/\mathrm{N}] & \text{if } 0<N(t_h)\leq N/2 \\ 1-[(2N(t_h)-N)/N] & \text{if } N/2<N(t_h)\leq N \end{cases}$$

If we simulate the micro behaviour by some experiment that generates *random numbers* for each element, we can observe that the combinatory system presents a chaotic macro behaviour, independently of the initial random impulse that shapes the initial states of the agents as shown in Figs. 6-9).

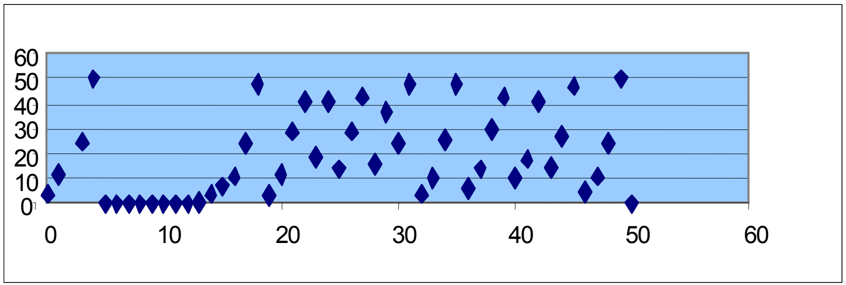


Fig. 6. Time Series of the Dynamics of a reversible probabilistic combinatory system of diffusion with N = 50 and showing chaotic macro behaviour. Number of iterations T=50 with neighbouring effect Test [A] – N(0) = 4 Probability increases straight line to 1 for N = 25 and then decreases to 0 for N=50.

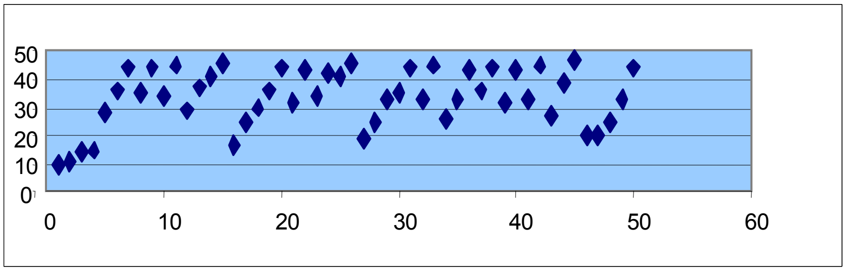


Fig. 7. Time Series of the Dynamics of a reversible probabilistic combinatory system of diffusion with N = 50 and showing chaotic macro behaviour. Test [B]-Changing initial value and keeping the same random numbers - N(0)= 5.

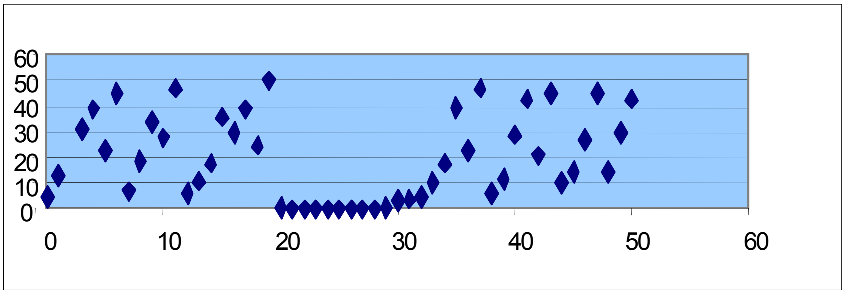


Fig. 8. Time Series of the Dynamics of a reversible probabilistic combinatory system of diffusion with N = 50 and showing chaotic macro behaviour. Test [C]-Probability increases straight line to 1 for N=40 and then decreases to 0 for N=50.

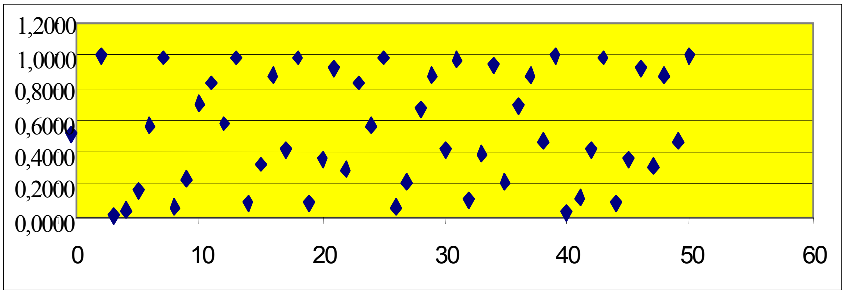


Fig. 9. Time Series of the Dynamics of a reversible probabilistic combinatory system of diffusion with N = 50 and showing chaotic macro behaviour. [D] - Dynamic system x = c x (1-x) with c= 3.99 and x0 = 0.85 for 50 iterations.

It is striking to observe how the random dynamics of the combinatory system shown in Figs. 6-8 are analogous to the chaotic behaviour shown by the simple quadratic function in Fig. 9 and, in particular, to the effects of path dependence.

THE HEURISTIC VALUE OF THE COMBINATORY AND SIMPLEX SYSTEMS APPROACH

The Study of Collectivities and the Sciences of Complexity

The Macro Approaches (a Short Survey)

Collectivities have always been a very complex subject of study, and for this reason a fascinating and interesting one as well. Originally, the study of collectivities considered as systems of agents followed the traditional *macro* or *analytic* approach, which produces a macro description of the behaviour of collectivities following only general macro rules and ignoring the micro behaviour of the agents.

Within the Sciences of Complexity the macro approach is typical of Population Dynamics Models, which try to represent population behaviour (increase, evolution, co-evolution and competition) in terms of the number of their elements, using, for example, Malthusian models and Volterra-Lokte equations in various forms (Ardeni & Gallegati, 1999; Volterra, 1931).

Wiener’s Cybernetics (Haken, 1977; Kauffman, 1993; von Foerster, 1960; Wiener, 1948) and, in particular, Evolutionary Cybernetics (Campbell, 1960; Gould, 2000), are other macro approaches which aim to explain how collectivities are able to arrange their components to form patterns different or better than the previous ones.

Von Bertalanffy's General System Theory (von Bertalanffy, 1968) and Haken's Synergetics (Haken, 1977), Forrester's Systems (Industrial) Dynamics approach (Forrester, 1961), Senge's System Thinking approach (Senge, 1990), and Maturana's and Varela's Autopoiesis approach (Maturana & Guiloff, 1980, Maturana & Varela, 1980, Varela, 1979, 1981; Zeleny, 1981) offer powerful conceptual frameworks and practical tools for building models of the behaviour of collectivities.

Collectivities as Complex Systems
The Micro Approaches (a short survey)

Conway's discovery of the fantastic world of *Life* (Gardner, 1970), Schelling's (1971) model of neighbourhood segregation and Sakoda's (1971) model of group formation are fundamental milestones in the study and the simulation of the behaviour of collectivities, focusing attention on the agent's micro behaviour and on local information; that is, following micro or internal or synthetic approaches.

The following, as the much famous *Micromotives and macrobehaviour* by Schelling (1978), is an attempt to offer through game theory and the prisoner's dilemma model a logical explanation for the *collective macro behaviours* shown by intelligent agents that, acting on behalf of their own interest, produce emergent collective dynamics.

In Agent-Based Models, collectivities are normally interpreted (Flake, 1998) as Complex (Adaptive) Systems (Allen, 1997; Axelrod, 1997; Coveney & Highfield ,1995; Goldspink, 2000; Jantsch, 1980; Mitleton-Kelly, 1997), defined as dynamic systems constituted by a plurality (usually large) of blind (reactive) or intelligent (active) multi-character (Drogoul & Ferber, 1994; Minsky, 1987), specialized, usually (strongly) interrelated, interconnected (Granovetter, 1974; Grimmett, 1999; Newman, 2003) and interacting agents (or processes) (Gell-Mann, 1995; Holland, 1975; Stacey, 1995), often showing possible multi-level hierarchies (Chan, 1998; Cummings & Staw, 1985; Gaffeo, 1999) whose collective macro behaviour is determined by the interaction of the micro behaviours of the agents (Otter, Veen & Vriend, 2001) on the basis of simple local rules (Waldrop, 1993) according to a schema, innate or learned (di Primio, 1999; Dooley, 1997), and which shows non-linear dynamics (Lewin, 1992) - so that the system's history is irreversible and the system's future unpredictable, or even chaotic, if the description of regularities is impossible – in the sense of Gell-Mann (1995), Wolfram's classification scheme (1984, 1994), or Devaney (1989) – as well as unanticipated global properties, or patterns (Foster & Metcalfe, 2001 Checkland, 1981).

The Complex Adaptive Systems approach, in particular Allen (1997; Kauffman, 1993; Holland, 1995), studies how collectivities interact and exchange information with their environment to maintain their internal processes over time through adaptation, self preservation, evolution and cognition (in the sense of Maturana & Varela, 1980: 13), and to achieve collective decisions (Rao, Georgeff & Sonenberg, 1992; Wooldridge & Jennings, 1994) within a relational context of micro behaviour (Conte & Castelfranchi, 1994; Gilbert, 1996).

The analysis of complex systems implies a Recursive Approach, and two of the most powerful tools are represented by the Cellular Automata Theory – introduced in the late 1940's by John von Neumann (Burks, 1966; von Neumann, 1966), which allows the researcher to explore complex systems by simulating Artificial Life (Liekens, 2000) – and the Genetic Algorithms approach (Bak, 1994, 1996; Schatten, 1999).

The theory of Cellular Automata builds mathematical models of a system whose agents are represented by cells in an array (a lattice) of one or more dimensions (Creutz, 1996; Schatten, 1999). It is important to note that the rules that define the micro behaviour of a cell are only local rules, in the sense that the state of the cell depends only on local information deriving from a specified number of neighbours and not on the state of the array (Dewdney, 1989, 1990; Gardner, 1970; Toffoli & Margolus, 1987; Ulam, 1991; Ulam, Reynolds & Rota, 1986).

Following the logic of cellular automata, many fundamental instruments have been created to simulate Artifical Societies (Epstein & Axtell, 1996; Heitkötter, 2004; Resnick, 1994). Among the most well-known are Resnick (1994) Termites and Dorigo's Ants approach (Dorigo, Di Caro & Gambardella, 1999; Hölldobler & Wilson, 1990), Langton's (1989) Swarm approach (Swarm Development Group, 1999), Reynolds's boids (Reynolds, 1987), and Dolan's Floys approach (1969; see also: Dolan, 1998).

These instruments also demonstrate that there is also a hidden order in the free behaviour of collectivities of simple living autonomous reactive agents obeying to simple logical rules. As Holland attempts to demonstrate, the most powerful approach to understanding and showing the hidden order in collective behaviour is the genetic algorithms approach (Holland, 1975) and the related genetic programming approach of Koza (Goldberg, 1989; Koza, 1992).

Koestler's *holonic systems* approach represents a different approach with respect to Agent-Based Systems (Koestler, 1968; Merli & Wheeler, 1995; Shimizu, 1987; Wilber, 2000), particularly useful for studying the behaviour of living organisms and social organizations.

These are composed of self-reliant units that are capable of flexible behaviour. More specifically, a holon can be thought of as a special type of agent that is characteristically autonomous, cooperative and recursive, and that populates a system or a collectivity. Holons form Holarchies, defined as hierarchically organized structures of holons.

In a Holarchy each Holon could be regarded as either a whole or as a part, depending on how one looks at it. A Holon will look as a whole to those parts beneath it in the hierarchy, but it will look as a part to the wholes above it. Thus a Holarchy is a whole that is also a structure of parts that are in themselves wholes.

Simplex Systems vs Complex Systems

Combinatory Systems constitute a particular class of Complex Systems but differ from *complex systems* and, in particular, from *complex adaptive systems* (CAS) and from Holarchies in many aspects.

Firstly, because *Combinatory Systems* do not necessarily present phenomena of adaptation (Gell-Mann, 1994) but, generally, some form of *self-organization* due to the *micro-macro feedback*, which is the simple adaptation of agents to a synthetic variable produced by the macro behaviour of the system. Adaptation may be a characteristic of some particular class of Combinatory Systems representing populations and not, in general, of collectivities conceived in a broader sense.

A *second* difference is observable also as regards the similarity of the agents: *"Here we confront directly the issues, and the questions, that distinguish CAS from other kinds of systems. One of the most obvious of these distinctions is the diversity of the agents that form CAS. Is this diversity the product of similar mechanisms in different CAS? Another distinction is more subtle, though equally pervasive and important. The interactions of agents in CAS is governed by anticipations engendered by learning and long-term adaptation."*. (Holland, 1995, p. 93).

The *third* main difference regards the absence of interactions among the agents (Lewin, 1992); in Combinatory Systems agents generally interact only with the macro variable characterizing the system and not with each other.

Finally, the theory of CAS observes the macro effects of the system produced by the agents that follow a schema or change the schema previously followed. Any *micro-macro feedback* between the micro behaviours and the schema is considered as a relevant characteristic. *"Schema define how a given agent interacts with other agents surrounding it. Actions between agents involve the exchange of* *information and/or resources. These flows may be non-linear. Information and resources can undergo multiplier effects based on the nature of interconnectedness in the system."* (Dooley, 1996, p. 2). For a synthesis, see Table 1.

Table 1. How do Simplex Systems differ from Complex Systems?

Complex systems and Holarchies	**Complex Adaptive systems**	**COMBINATORY SYSTEMS**
Agents are heterogeneous	Diversity of the agents as a constitutive feature	Agents are similar
Agents are interconnected and show hierarchy	Agents are interconnected	Agents are not interconnected
Micro behaviours are differentiated	The Agents present phenomena of adaptation	Micro behaviours are analogous
Agents act following local rules	Agents act following a schema	Agents act following the *micro-macro feedback*
Decisions are prevalently based on the prisoner's dilemma schema	Decisions are based on forecast and expectations	Decisions follow a simple one column pay-off matrix

If the micro behaviours of the agents are determined exclusively by the macro behaviour, then the Combinatory Systems is a *pure simplex system*. If they also depend on an opportune neighborhood as well as, naturally, the macro behaviour, the simplex system is characterized by complete and *limited information*.

Finally, if the agents' behaviour depends only on local rules acting on a defined neighborhood, without considering any micro-macro feedback, the system is a *Complex System* that obeys local and *limited information*, loses the characteristics of a simplex system and can be simulated by traditional *cellular automata*.

CONCLUSIONS: EXPLORING COLLECTIVITIES THROUGH THE COMBINATORY SYSTEM THEORY

The *Theory of Combinatory Systems* (Mella, 2001) proposes models to interpret the collective phenomena and searches for the conditions that produce the macro behaviours. In particular, the theory focuses on the necessity both of recognizing the nature of the *global variables* that act as global information and of understanding the nature of the *macro rules*, which specify the *recombining factor(s),* and the micro rules, which specify the *necessitating factor(s);* The joint action of these factors gives rise to and maintains the macro and micro behaviours.

Three aspects of this theory make it particularly effective: (a) It is not limited to describing the macro behaviour of the collectivity based on general rules or the agent's behaviour based only on local rules, but tries to uncover and explain above all the *feedback* between the macro and micro behaviours or their effects; it is *neither* a *macro* approach, since it also refers to local rules by considering micro behaviours, nor a micro approach, since it also includes the macro behaviour in the model of the *system*; it is rather a *micro-macro* approach, precisely in that the *operating rules*, describing the behaviour of the system, must in some way include not only *local rules* but also the *feedback* between the micro and macro behaviours (Rousseau, 1985; House, Rousseau & Thomas-Hunt 1995); (b) to understand the phenomena attributable to the action of Combinatory Systems the theory tries to uncover and make clear the *necessitating factors* (that cause the micro behaviour of each agent in the system) and the *recombining factors* (that produce and maintain the unit's macro behaviour); the theory then concludes that, in the presence of suitable necessitating and recombining factors, "chance" will trigger the dynamic process of the system that "by necessity" is then maintained and influences the individual behaviours; (c) the procedural explanation offered by the theory not only allows us to understand the operating mechanism that produces the phenomena under examination, but also permits us to determine the most effective forms of control.

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