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Combinatory Systems and Automata: Simulating Self-Organization and Chaos in Collective Phenomena

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Abstract: In plain words, I define as (social) Combinatory Systems a particular class of unorganized systems made up of a collectivity of similar agents (not functionally specialized, not necessarily interconnected by evident interactions) each of which is capable of producing a micro behaviour, and a micro effect, analogous to that of the others. If, on the one hand, the macro behaviour of the System, as a whole, derives from the combination – appropriately specified (sum, product, average, min, max, etc.) – of the analogous behaviours (or effects) of its similar agents (hence the name Combinatory System), on the other hand the macro behaviour (or the macro effect) represents a global information that determines, or conditions, or directs, by necessity, the subsequent micro behaviours. A Combinatory Automaton is a simple tool to simulate combinatory systems. This is composed of a lattice, each of whose cells contains a variable representing the state of an agent. The value of each cell at time th depends on a synthetic global variable whose values derive from some operation carried out on the values of the cells and that represents the synthetic state of the automaton. The micro-macro feedback connects the analytical values of the cells and the synthetic state of the automaton. I will try to demonstrate that combinatory systems represent a wide range of the behaviours of collectivities, that Combinatory Automata are a powerful tool for simulating the most relevant combinatory systems, and that combinatory systems, despite their simplicity, can show chaotic dynamics and, of course, path dependence.

Keywords: Agent-based Systems, Combinatory System, Combinatory Automaton, Populations and Collectivitiesù, Chaos in Social Behaviour

The Study of Collectivities

I **DEFINE** *collectivity* (social system, population, collective unit, social totality, group, plurality, collection, matrix, and so on) as a set of elements, or agents, that produce *individual micro* behaviours (which lead to *micro* effects of some kind), but which, as a whole, produce a *macro* behaviour (and at times a *macro* effect or a recognizable pattern) which is not included in advance in the operating programme of the agents' behaviour.

If considered *from a certain distance* collectivities appear distinct with respect to the individuals, and thus seem able to produce an autonomous *macro* behaviour due to the interactions of the *micro* behaviours. This *macro* behaviour may show a chaotic dynamic or a regular one as a result of some kind of self-organization.

Collectivities have always been a very complex subject of study, and for this reason both fascinating and interesting.

Since Thomas Schelling's attempt in his very famous work, *Micromotives and Macrobehavior*, to offer through game theory and the prisoner's dilemma model a logical explanation of why collective *macro* behaviour derives from the *micro* behaviours of intelligent agents (Shelling, 1978), and Conway's discovery of the fantastic world of Life (Gardner, 1970), the study and simulation of the behaviour of collectivities or of agents has followed *micro* or internal or synthetic approaches.

The Complex Adaptive Systems approach, in particular (Gell-Mann, 1995), studies how collectivities interact and exchange information with their environment to maintain their internal processes over time through adaptation, self preservation, evolution and cognition.

The analysis of Complex Systems implies a *recursive approach*, and two of the most powerful tools are represented by the Cellular Automata Approach – introduced in the late 1940´s by John von Neumann (Burks, 1966), which allows the researcher to explore complex systems by simulating Artificial Life (Alife) (Liekens, 2000) – and the Genetic Algorithms approach (Bak, 1996; Schatten, 1999).

The Cellular Automata approach builds mathematical models of a system whose agents are represented by cells in an array (a lattice) of one or more dimensions (Creutz, 1996). It is important to note that *the rules that define the micro behaviour of a cell are only local rules, in the sense that the state of the cell depends only on one of a specified number of neighbours and not on the state of the array* (Toffoli and Margolus, 1987; Dewdney, 1990; Ulam, 1991).

Towards Combinatory Systems

Concentrating on the *recursive approaches*, I observe that if, on the one hand, it is easy to explain (perhaps properly speaking, to describe), assuming only local rules, the behaviour of a flock of birds, a school of fish, or a herd of elephants when these collectivities have already formed, or the spread of information, the imitation of choices (information contagion), or the percolation effects in probabilistic diffusion systems (Grimmet, 1999), on the other it is not so easy to apply this *micro* approach to describe, for example, the grouping of flocks (a bird is attracted by the flock and not by its neighbours), swarms, herds and other collectivities, the formation of graffiti on walls (people are attracted by the cloud of graffiti and not by the behaviour of other people), the breaking out of applause (many people applaud if the applause increases), or the phenomenon of a rising murmur in a crowded room.

It is clear that a tower was built by a medieval noble family from Pavia (see below) not only after observing the neighbourhood but the whole swarm of towers in the town as well. It is also clear that a person who is talking raises his voice above the increasing murmur of the crowded room only because of individual necessity and not because those around him are raising their voices; the applause begins, rises and is maintained because the clapping itself directs the clapping people; or that a fish joins a school of fish because of the presence of a predator, and only if he can perceive the school, not because he sees other fish join the school.

In many cases, moreover, agents cannot observe the collectivity, and thus their neighbours, and must act only based on individual necessities, as in the case of the formation of piles of garbage (if I need to throw away a piece of garbage and I see a garbage pile, I prefer to add my garbage to the pile), of annoying and dangerous wheel ruts on the highway (passing trucks need to maintain their trajectory on the carriageways, and these are reinforced by these *micro* behaviours), or of paths in fields (people prefer to cross a field where a path is visible), and so on.

How do we explain the formation of paths in fields? What is the force behind the continual improvement in the quality of products? How does a feud develop? Why are some park benches or walls covered by graffiti while others nearby are spotless? Why are records continually broken? What mechanism can we use to explain the maintenance of languages and dialects in limited areas?

In all these circumstances, the Agents' *micro* behaviour seems to follow some necessitating *macro* variable(s), or *global information*, deriving from the collectivity (the cloud of graffiti, the pile of garbage, the applause, the carriageway, the feud, and so on) rather than obey a set of *local rules.*

I think that these and many other interesting phenomena, or effects, might be attributed to the basic behaviour of very simple collectivities that I have called Combinatory Systems – or Simplex Systems – since they represent a particular class of systems acting in a "combinatory" way[1].

The Combinatory Systems Theory

A brief Overview

In plain words, I define as (social) *Combinatory System* (Mella, 2005) a collectivity of *similar agents* (not functionally specialized and not necessarily interconnected by evident interactions) each of which is capable of producing a *micro behaviour* and a *micro effect analogous* to that of the others. Combined together the *micro* behaviours and effects produce a *macro behaviour* and a *macro effect* attributable to the *collectivity* (fig. 1).

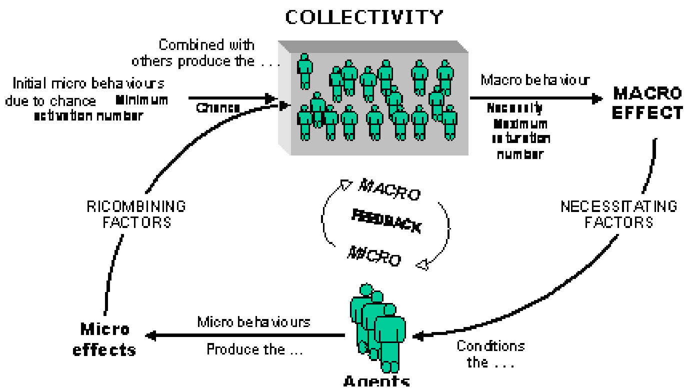


Fig.1: Qualitative model of a Combinatory System

If, on the one hand, the *macro* behaviour of the System as a whole *derives* from the *combination,* appropriately specified (sum, product, average, min, max, etc.), of the analogous *micro* behaviours (or effects)

[1] The Theory of *Combinatory Systems* and the related bibliography are at the site: www.ea2000.it/cst.

of its similar agents (hence the name Combinatory System), on the other hand the *macro* behaviour (or the *macro* effect) *determines*, or *conditions*, or *directs* the subsequent *micro* behaviours (Fig. 2).

This internal *micro-macro feedback* between *micro* and *macro behaviours* – or between their *micro* and *macro effects* – guarantees the maintenance over time of the system's *micro* and *macro* dynamics[2].

The *macro behaviour* – or its *macro* effects – may be thought of as *global information* (the applause, the cloud of towers, the school of fish, the cluster of firms composing an economic district, and so on) deriving from the collectivity, or as an *internal organizer* which is produced by the agents and modifies their *micro* behaviour over time.

If the *micro* behaviours of the agents are determined exclusively by the *macro* behaviour, the combinatory system is a *pure combinatory system*.

If they also depend on an opportune *neighborhood* as well as, naturally, on the *macro* behaviour, the combinatory system is characterized by *limited information*.

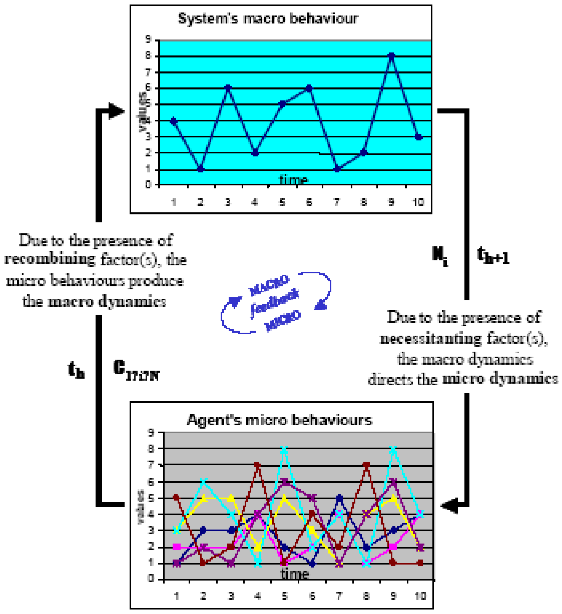


Fig. 2: The Micro and Macro *Dynamics* and the Micro-Macro *Feedback*

Since by definition the agents are similar and show similar behaviour, it follows that we can assume that the same *global information* produces *similar* decisions regarding the change in state of the agents, who thus appear to conform or even *synchronize* their *micro* behaviours and to produce interesting forms of self-organization (Jantsch, 1980), or spontaneous order (Sugden, 1989; Ashford, 1999).

When the system starts up "by chance" it then maintains its behaviour "by necessity", as if an Invisible Hand or a Supreme Authority regulated its time path and produced emerging phenomena, observable effects and patterns.

There is nothing metaphysical here: the *invisible hand* is nothing other than the *effect* of the *micro-macro feedback* action that generates and updates the *global information* that produces *synchronization*, *self-organization* and emerging *macro* behaviours attributable to the collectivity.

Necessitating and Recombining Factors

The feedback arises from *necessitating factors* and is maintained by the action of *recombining factors*.

Necessitating factors are the factors that *force* the agents to *adapt* their *micro* behaviour to the system's global information (*macro* behaviour or effect). They may be a constraint, a rule, a condition, a law, a conviction, an imitative act, a biological or social impulse, etc., and result from obligation, imitation, convenience, utility, desire, etc., and depend on the culture, education, distinctive mental and emotional conditions, etc., of the individual agents.

Recombining factors are the factors that *allow* the system to *notice* and *recombine* the *micro* behaviours (or the *micro* effects) in order to produce and maintain the *macro* behaviour (or the *macro* effect). They may derive from a rule, a convention, an algorithm, etc., and may also simply follow on from the conditions of the environment, or result from the social

[2] *Combinatory Systems* (CS) differ from *Complex Adaptive Systems* (CAS) in many respects, in particular because *Combinatory Systems* do not necessarily present phenomena of adaptation but generally some form of self-organization due to the *micro-macro feedback.*

condition or the culture of the collectivity constituting the system.

Recognizing the existence of a micro-macro feedback and understanding the nature of both the necessitating *factors and the* recombining *ones is indispensable for interpreting collective phenomena as deriving from a combinatory system* (for simplicity's sake I have not considered energy inputs)..

When not caused by external decisions, the activation of a combinatory system normally needs an *initial casual input*: the agents must produce a sufficient number of *micro* behaviours greater than a *minimum activation number*, or a minimum density, and lower than a *maximum saturation number*, or a maximum density.

Once the minimum density is reached "by chance", and if opportune sets of necessitating and recombining factors are present, the *micro-macro* feedback guarantees that the *macro* behaviour "by necessity" initiates and grows, feeding on the subsequent *micro* behaviours and, at the same time, conditioning them.

Typology of Combinatory Systems

The logic proposed in the previous sections can be observed in four relevant classes of combinatory systems which differ with regard to their *macro* behaviour (or their *macro* effect).

1. Systems of *accumulation*, whose *macro* behaviour leads to a *macro* effect which is perceived as the accumulation or the clustering of "objects", behaviours, or effects of some kind; this logic applies to quite a diverse range of phenomena, among which the formation of urban or industrial settlements of the same kind and of industrial districts, the grouping of stores of the same type in the same street (Mella, 2006), the accumulation of garbage, graffiti, writings on walls; but it can also be applied to phenomena such as the breaking out of applause, the formation and the maintenance of colonies, forests, herds and schools.

2. Systems of *diffusion*, whose *macro* effect is the diffusion of a trait or particularity, or of a "behaviour" or "state", from a limited number to a higher number of agents of the system; systems of diffusion explain quite a diverse range of phenomena: from the spread of a fashion to that of epidemics and drugs; from the appearance of monuments of the same type in the same place (the Towers of Pavia, for example, whose dynamics is simulated in section 3.2) to the spread and maintenance of a mother tongue, or of customs.

3. Systems of *pursuit* produce a behaviour that consists in a gradual shifting of the system toward an "objective", as if the system, as a single entity, were pursuing a goal or trying to move toward increasingly more advanced states; this model can represent a lot of different combinatory systems: from the pursuit of records of all kinds to the formation of a buzzing in crowded locales; from the start of feuds and tribal wars in all ages to the overcoming of various types of limits.

4. Systems of *order* produce a *macro* behaviour, or a *macro* effect, perceived as the attainment and maintenance of an ordered arrangement among the agents that form the system; systems of order can be used to interpret a large number of phenomena: from the spontaneous formation of ordered dynamics (for an observer) in crowded places (dance halls, pools, city streets, etc.) to that of groups that proceed in a united manner (herds in flight, flocks of birds, crowds, etc.); from the creation of paths in fields, of wheel-ruts on paved roads, of successions of holes in unpaved roads, to the ordered, and often artificial, arrangement of individuals (stadium wave, Can-Can dancers, Macedonian phalanx).

5. Systems of *improvement and progress*, whose effect is to produce progress, understood as an improvement in the overall state of a collectivity that is attained through individual improvement.

Individual improvements raise the parameter that measures collective progress; this leads to the formation of positive and negative gaps that push the individuals to improve in order to increase the gaps (if positive) or eliminate them (if negative). The system must be able to notice the individual improvement and to adjust the progress parameter to the average (or, more generally, to the combination) of the individual improvement measures.

Combinatory Automata

Combinatory Automaton

To understand collective phenomena following the Combinatory System perspective, it is useful to build a Combinatory Automaton that specifies the mathematical and statistical simulation model that represents the behaviour of the Combinatory Systems.

As we know, a cellular automaton can be represented by a grid whose cells represent an agent of the system that can take on a particular determinate set of states. Each agent (cell) changes its state according to *local rules*, which derive from the state of its neighbours, based on a given convention. The overall *macro* state of the cellular automaton at a given time "t" is represented by the state of all its cells at "t", and it evolves step by step in relation to the *macro* dynamics of the cells.

Since in Combinatory Systems the agents operate on the basis of *global information* from the combination of the micro behaviour of all the other agents, a (two-dimensional) Combinatory Automaton must possess the following characteristics (fig. 3):

1. it is made up of a grid of **N**=(RxC) cells, each of which represents an agent, A_{ij}, $1 \leq i \leq R$, $1 \leq j \leq C$ (in a mono-dimensional Automaton the agents are arranged in a vector of **N** cells);
2. each agent, A_{ij}, is characterized by a *micro* state, $x_{ij}(t_h)$, for every $t_{h\varepsilon T}$, h=0, 1, 2, … ;
3. each *micro* state may produce an analogous *micro* effect, $e_{ij}(t_h)=f_{ij}[x_{ij}(t_h)]$, which constitutes an observable output of A_{ij};
4. the set of the *micro* states or effects represents the *analytical state* of the system;
5. the agents' *micro* states are combined together to determine a *macro* state: $Y(t_h)=C\, x_{ij}(t_h)$ attributable to the system as a whole; the function C represents the set of recombining factors;
6. the *macro* state may produce a *macro* effect $E(t_h)=F[Y(t_h)]$ which constitutes an observable output of the system and represents *global information* for each agent in order that it may modify its state;
7. the subsequent *micro* states are a function of the *macro* state (Moore automaton), of a *transition of state probability* $p_{ij}(t_h)$, and, in many cases, of the previous *micro* state as well (Melay automaton): $x_{ij}(t_{h+1})=N_{ij}\,[Y(t_h), x_{ij}(t_h), p_{ij}(t_h)]$, where N_{ij} represent a set of *necessitatingoperation(s)* which modifies the previous values, $x_{ij}(t_h)$; if the $p_{ij}(t_h)$ are not specified, the Combinatory Automaton is deterministic;
8. the Combinatory Automaton presents an evident *micro-macro feedback,* since the *micro* behaviour of an element depends on the *macro state* of the Automaton; but this in turn derives from the combination of the *micro* states of the cells (the *analytical state* of the automaton);
9. an initial *analytical state* (random or programmed) is necessary;
10. the rules that specify the initial *analytical state*, the functions C and N_{ij} as well as the probabilities $p_{ij}(t_h)$, form the operational programme of the Combinatory Automaton.

Probabilistic Combinatory Automata

The most interesting Combinatory Automata are the probabilistic ones that seek to examine the dynamics of the state of the Combinatory System – and thus its *macro* behaviour and *macro* effect (if these two elements do not coincide) – by considering the *probability* that the individual elements of the grid will change their state and trigger the system's dynamics.

We assume a Probabilistic Combinatory Automaton is composed of three matrices:

1. the *matrix of the states* of the agents, written as values for the x_{ij} of each cell; these values are updated at each subsequent moment according to the functions N_{ij};
2. the correlated *matrix of the probability of transition* of state; that is, the probability field that at each moment indicates the probability of a change of state for each cell;
3. the correlated *matrix of the periods of transition* of state that at each moment indicates the length of the *period of transition* of state (assumed constant over time and equal for each cell; this assumption is not unrealistic, since it is typical of a wide range of combinatory systems).

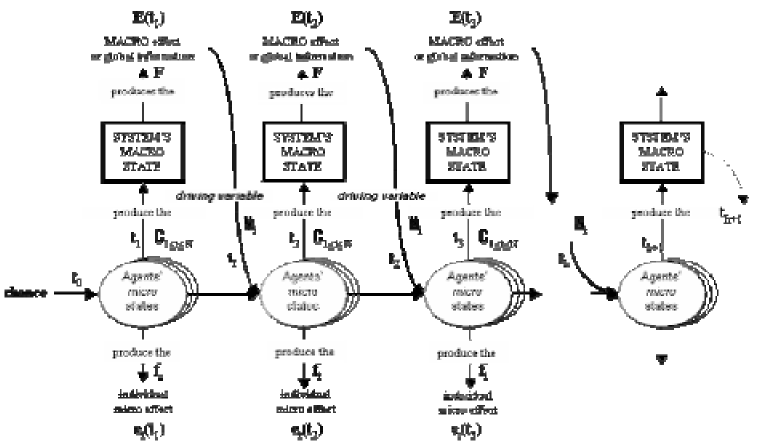


Fig. 3: The Dynamics of a Combinatory Automaton

Example of probabilistic Irreversible Combinatory Automata simulating Diffusion and Accumulation

Let us consider a Combinatory Automaton composed of **N**=100 elements arranged in a (10x10) square matrix which admits only two states, **a**="**1**" and **b**="**0**".

At time t=0 all the elements are in state "**0**", as shown in the left matrix in fig. 4(1). The right matrix in fig. 4(1) represents – in hundredths – the constant probabilities **p** $_{ij}(t_0)$=1/100 of transition of state for each $A_{ij}(t_0)$ (the symbol %, has been omitted).

To simplify the example, I have assumed that the **p** $_{ij}(t_h)$ are functions of the number $0 \leq E(t_h) \leq$ **N** indicating the *synthetic state* of the system at time "t_h" – that is, the number of elements showing the state "**1**" in the lattice.

We also assume that the Combinatory Automaton must simulate a *limited-information system* according to which the micro behaviour of each element depends *not only on the global information, E*(t_h), but also on the local information concerning the micro behaviour of a *convenient neighbourhood* of A_{ij}.

We can now use a random numbers table (not shown here) to simulate the behaviour of the system after having identified an element that undergoes *the initial impulse* and thus "by chance" changes its own state.

Fig.s 4(2) show the new matrices after four iterations.

Combinatory automaton at t=0 $E(0)$ = **0**

	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0

Field of probabilities (%) $E(0)$ = **0**

	1	2	3	4	5	6	7	8	9	10
0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1

Fig. 4(1): Combinatory Automaton simulating diffusion - Stage 0

Even if the example seem simple, it nevertheless can represent a great number of Combinatory Systems of diffusion and, in particular any form of human settlement or horizontal cluster; we need only assign to state "**0**" the meaning of "no settlement" and to state "**1**" that of "settlement". A similar interpretation holds for the formation of industrial clusters and industrial districts. The presence of a certain number of firms in a given site increases the probability of other settlements (Mella, 2006).

Combinatory automaton at t=4 $E(4)$ = **13**

	1	2	3	4	5	6	7	8	9	10
0	0	0	0	1	0	0	0	1	0	0
1	0	0	1	0	1	0	0	0	0	0
2	0	0	0	1	0	0	0	0	0	0
3	0	0	1	1	1	0	0	0	0	0
4	0	0	0	0	1	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	1	1	0	0
9	0	0	0	0	0	0	1	0	0	0

Field of probabilities (%) $E(4)$ = **13**

	1	2	3	4	5	6	7	8	9	10
0	2	6	11	100	11	6	6	100	6	2
1	2	6	100	21	100	6	6	6	6	2
2	2	11	21	100	21	11	2	2	2	2
3	2	6	100	100	100	11	2	2	2	2
4	2	6	11	21	100	11	2	2	2	2
5	2	2	2	6	6	6	2	2	2	2
6	6	6	2	2	2	2	2	2	2	2
7	100	6	2	2	2	6	11	11	6	2
8	6	6	2	2	2	11	100	100	6	2
9	2	2	2	2	2	11	100	16	6	2

Fig. 4(2): Combinatory Automaton simulating diffusion - Stage 4

The easiest way to construct a simple irreversible Combinatory Automaton simulating *accumulation* is to assume that each cell represents an agent which, at every iteration, accumulates as a function of the system's synthetic state $E(t_h)$, which modifies the **p** $_{ij}(t_h)$ for each agent. The rule of transition of state is: if $A_{ij}(t_h)$="**n**" and the event occurs to which probability **p** $_{ij}(t_h)$ is associated, then it becomes $A_{ij}(t_{h+1})$="**n+1**"; otherwise it remains in state "**n**".

Fig. 5 shows the accumulation effect produced by a probabilistic irreversible Combinatory Automaton composed of 100 agents after 50 iterations.

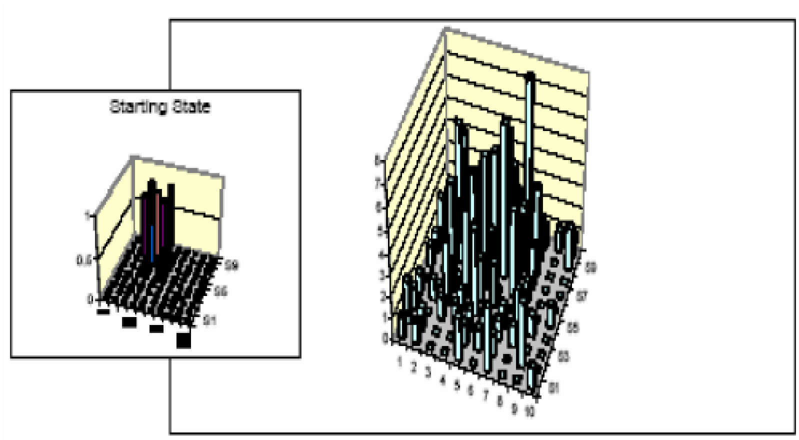


Fig. 5: An Irreversible Combinatory Automaton of Accumulation

The Combinatory Automaton of *accumulation* presented in fig. 5 can represent an explanatory model of many accumulation phenomena.

An emblematic exemple is the formation of garbage piles; a small accumulation of garbage that forms "by chance" leads to an increasingly larger accumulation, under the condition that the individuals who leave their garbage prefer to do so where there already is a pile.

The more the accumulation grows the greater the probability that more garbage will be dumped on that site; the formation of garbage piles becomes inevitable, as our own direct observation reveals.

Fig. 6 provides a clear example of the power of the Pile of Garbage Combinatory System. In the photo we can see a bicycle basket full of garbage left by the collectivity of pedestrians walking along a central street in Tokyo. The basket on the second bike is being filled.

The same model can depict any form of vertical cluster, since vertical clusters can be simulated by an irreversible Combinatory Automaton of accumulation in which the probabilities of the cells in which some element is already located increase with the number of elements in the cell, and the probability of the neighbouring cells increases with the number of elements in the grid (synthetic state).

Fig. 6: An Example of a Combinatory System of Accumulation

Chaos in Stochastic Combinatory Automata

In stochastic Combinatory Automata probabilities can act in two ways:

1. as *stop-or-go probabilities*, $\mathbf{p}_{ij} \equiv \mathbf{p}_{ij}[E(t_h)]_{[0,1]}$; "0" means that if the event does not occur, the agent maintains its state; "1" that the agent changes its state if the event occurs;

2. as *transition probabilities*, $\mathbf{p}_{ij} \equiv \mathbf{p}_{ij}[E(t_h)]_{[-1,1]}$; if the probabilistic event occurs, then the agent enters a new state; if the event does not occur, the agent assumes a different state or returns to the past one.

The social Combinatory Systems that are most interesting and easiest to represent are the *irreversible* ones (build a tower or not, teach Italian or English to babies). In these systems both the *micro* and *macro* behaviours produce permanent effects that may be viewed as increasing or decreasing cumulative processes regulated by *stop-or-go probabilities*.

Chaos arises in *combinatory systems* when the *hypothesis of reversibility* is introduced (for example: to speak or to keep quiet in the next minute, wear a skirt or miniskirt on different days, choose road A or B on different days).

These systems are generally governed by *transition* probabilities, $\mathbf{p}_{ij}[E(t_h)]_{[-1,1]}$, so that we admit that a cell could change its state "**0**" → "**1**" as well as "**1**" → "**0**" at different times; as a result the Combinatory Automaton might show a chaotic *macro* behaviour in the sequence of *macro* states $E(t_h)$, h=0, 1, 2, …

Let us assume, for example, that the probabilities take on the following values corresponding to a simple *tent map*:

$$p_i[E(t_h)]_{[-1,1]} = \begin{cases} \dfrac{2[E(t_h)]}{N} & if\ 0 < E(t_h) \le \dfrac{N}{2} \\ 1 - \dfrac{[2E(t_h) - N]}{N} & if\ \dfrac{N}{2} < N(t_h) \le N \end{cases}$$

As shown in fig. 7, this Combinatory Automaton presents a **"**chaotic" *macro* behaviour, so that the system's history is irreversible and the system's future unpredictable, since the description of regularities is impossible (Wolfram, 1994).

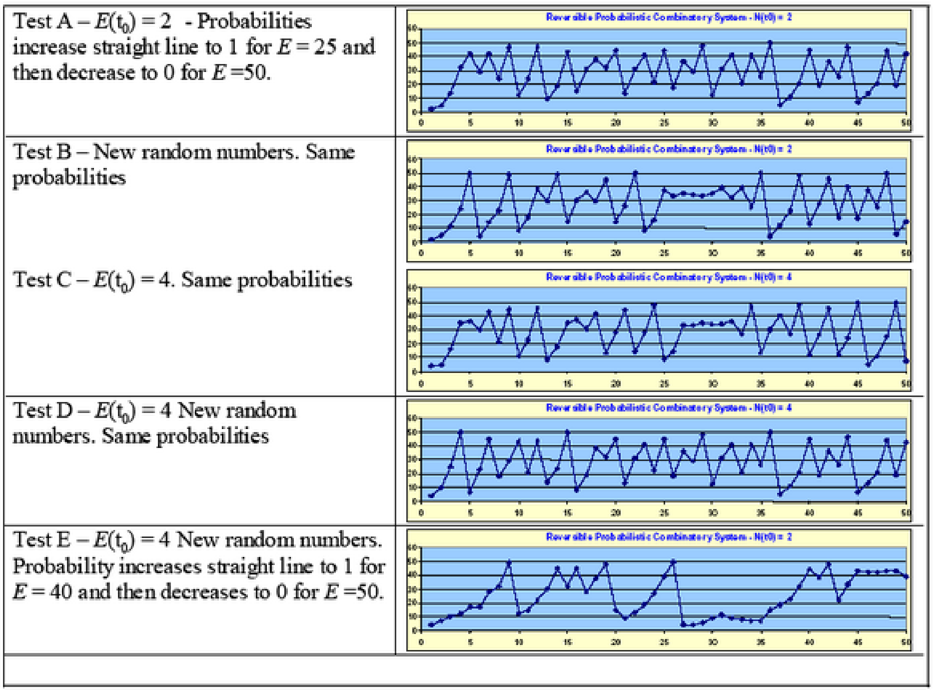


Fig. 7: Chaotic *macro* behaviour shown by a reversible Combinatory Automaton of diffusion with N=50 cells and T=50 iterations

Two Examples of Combinatory Automata

A Reversible Combinatory Automaton Simulating the Murmur and Noise in Crowded Rooms

The following verbal *heuristic model* illustrates the *rules* that give rise to the phenomenon of a murmur arising in a crowded room:

necessitating rule: if you have to talk and you hear a background murmur or noise, raise your voice level several decibels above the background noise;

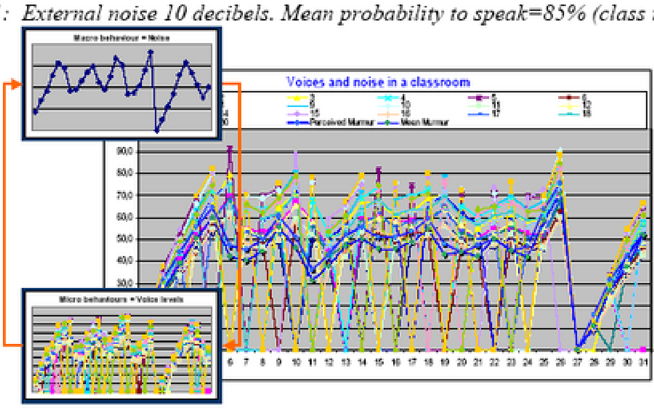
recombining rule: the environment preserves the noise, the collectivity makes interpersonal communication necessary or favours it; we can also take account of a parameter which represents the noise factor from causal factors which are different from the *micro* behaviours (bells ringing, shoutings from outside the system's environment, etc.);

micro-macro feedback: the individual, in order to be heard, must speak louder than the level of the background noise that results from the *macro* behaviour. The system produces a background noise which is a function of the *micro* behaviour of those who, in order to be heard, must speak in a loud voice.

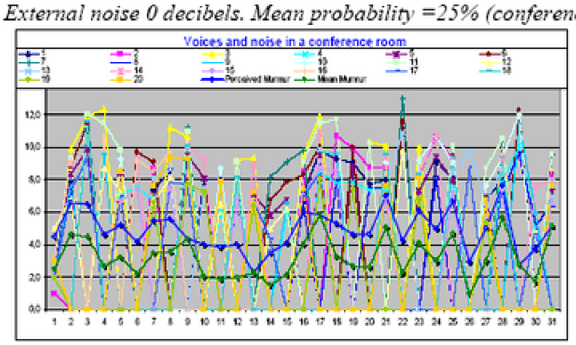
We can represent this system as a probabilistic reversible Combinatory Automaton in which each cell represents an individual speaker and the output of the cell expresses his voice level; the *murmur* is the *synthetic output* of the Automaton deriving from the mean value of the voice levels.

The talking agents represented by the cells thus seem self-*organized* to simultaneously raise their voice level and produce a stable, a rising, or a fluctuating noise: a typical pattern which, I am sure, we have all experienced on more than one occasion and in different locales, as shown in fig. 8.

Test 1: External noise 10 decibels. Mean probability to speak=85% (class room)



Test 2: External noise 0 decibels. Mean probability =25% (conference room)



Test 3: External noise 10 dec.s. Mean probability =3% (theater during a performance)

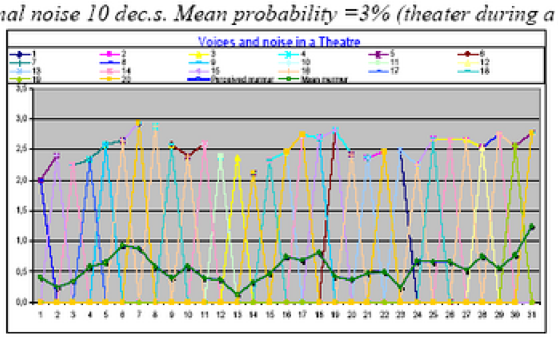


Fig. 8: Model of Murmur and Noise system with 20 Agents

An Irreversible Combinatory Automaton Simulating the Towers of Pavia

Pavia, the town where I was born, is a very nice town on the river Ticino, in northern Italy, which is not only famous for having the oldest University in the world, founded by King Lotario in A. D. 825, but also for its many towers, which explains why it was called Civitas turrigera, Civitas centum turrium, "the city of the hundred towers".

The tower phenomenon began around the year 1000 A.D., perhaps some decades before, and rapidly developed in the following centuries, so that around the year 1300 many had already been ruined.

In 1570 the historian Breventano mentioned more than 150 towers, a very large number if we consider that within its walls Pavia had a surface area of only 900 x 900 meters; Spelta (1603) counted around 100, and Zuradelli (1888) 76. In a fresco from 1525, more than 70 are countable (fig. 9). A recent map of the town (1965) reveals 71 traceable towers, 7 of them almost completely intact (fig. 10).

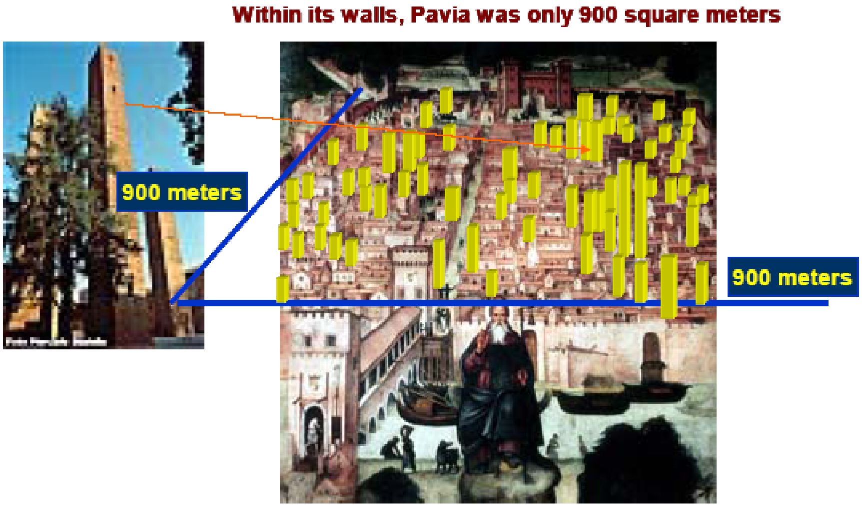


Fig. 9: The Towers of Pavia in a Fresco from A.D.1521

The towers of Pavia were built on the outsides of the palaces of important families to celebrate the birth of a male heir. They are on average 40 meters high, some even 50-60 meters in height, square, and with each side around six meters wide. They are built in brick and are windowless and without an interior; the towers are truncated, with a flat or slightly slanted roof just barely covering the tower.

These features make them very compact but slender, with a typical clay colour.

The towers of Pavia do not have a specific function; they are simply three-dimensional slender icons that from afar could indicate to visitors where the palace was located.

In order to explain this unique phenomenon I have built a Combinatory Automaton simulating a Combinatory System of diffusion, shown in fig. 11.

Fig. 10: The Towers of Pavia in a recent research commissioned by the Mayor of the Town in 1965

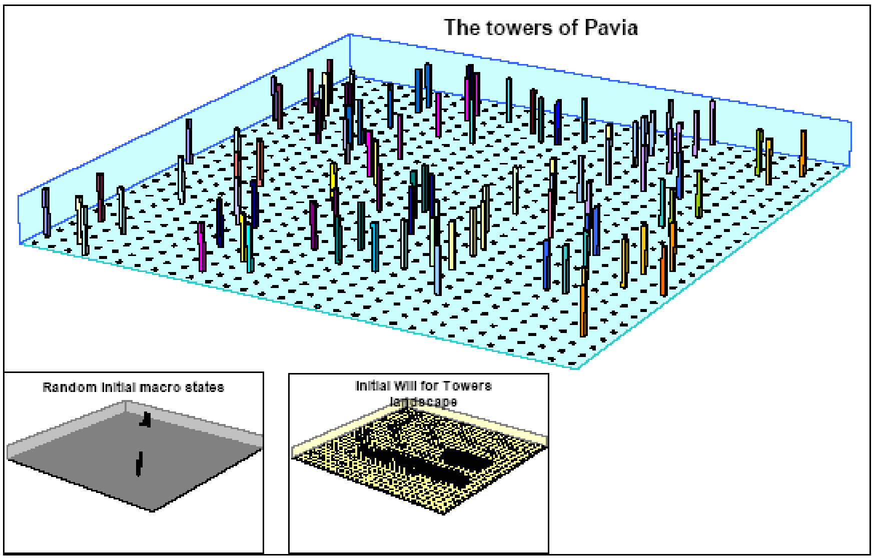


Fig. 11: The *analytical state* of the Combinatory Automaton simulating the Towers of Pavia in a square grid of 900 cells (the buildings of noble families) for 25 iterations (1 iteration equals 10 years)

The Combinatory Automaton is made up of a square grid of 900 cells (the buildings of noble families) for 25 iterations (1 iteration equals 10 years) in which each family (agent) at any discrete t_h shows only two states: "**1**=a tower" or "**0**=no tower".

The synthetic state of the automaton is represented by the variable $E(t_h)$, which indicates the number of towers and the *global information* which determines the field of probabilities for each cell to translate its state from "**0**" to "**1**", in the sense that the state of each A_{ij} depends on the probability a tower will be built, which in turn depends on the state of the system which defines the *macro* behaviour.

The maximum number of towers is 186 after 10 iterations. Figure 11 shows 98 towers after 25 iterations. The correspondence between the real data and that of the model is striking.

Conclusions and Challenges

Combinatory System Theory studies the collectivities of similar agents whose analogous *micro* behaviour produces a *macro* behaviour that refers to the collectivity as a whole; the *macro* behaviour of the collectivity produces a *macro* effect that represents global information that guides the subsequent *micro* behaviour, thereby producing forms of self-organization and synchronization, as well as forms of chaotic behaviour.

Combinatory System Theory focuses attention on the importance of both the *micro-macro feedback* and the *necessitating* and *recombining factors* that produce and maintain it.

The challenge of Combinatory System Theory is threefold: (i) to develop more general and further sophisticated Combinatory Automata for any specific class of combinatory system; (ii) to apply the theory to understanding collectivities operating in the real world; (iii) to specify, for any real observed collective phenomenon, the sets of necessitating and recombining factors which allow us to interpret and control the collectivity that produces it.

References

Ashford, N. (1999). Spontaneous Order. *The Freeman: Ideas on Liberty,* 49(7), at http://www.fee.org/freeman/99/9907/ashford.html.

Bak, P. (1996). *How Nature Works: The Science of Self-Organized Criticality.* Berlin: Springer.

Breventano, S. (1570). *Istoria della antichita nobilta, et delle cose notabili della citta di Pauia.* Hieronimo Bartholi Ed. S. Pietro in Ciel' Aureo, Pavia.

Burks, A. W. (1966) (Ed.). *Theory of Self-Reproducing Automata [by] John von Neumann.* Urbana: University of Illinois Press.

Creutz, M. (1996). *Cellular Automata and self organized criticality*, at: http://ttt.lanl.gov/PS_cache/hep-lat/pdf/9611/9611017.pdf.

Dewdney, A. K. (1990). *The Magic Machine*. New York: WH Freeman.

Gardner, M. (1970). Mathematical Games. *Scientific American,* 223(4), 120-123.

Gell-Mann, M. (1995). What is complexity? *Complexity*, 1(5), 16-19.

Grimmett, G. (1999). *Percolation* (2^{nd}. ed.). Berlin: Springer-Verlag.

Jantsch, E. (1980). *The self organizing universe Scientific and Human Implication: of the Emerging Paradigm of Evolution*. New York: Pergamon Press.

Liekens, A. (2000). *Artificial Life*. In A. Ralston, E. D. Reilly, & D. Hemmendinger (Eds.), *Encyclopedia of computer science* (pp. 93-96). London: Nature Publishing Group.

Mella, P. (2000). *Combinatory System Theory*. At: www.ea2000.it/cst.

Mella, P. (2005). Observing collectivities as simplex systems. The combinatory systems approach. *Nonlinear Dynamics, Psychology, and Life Sciences*, 9(2), 121-153.

Mella, P. (2006). Spatial Co-localisation of Firms and Entrepreneurial Dynamics. The Combinatory System View, *International Entrepreneurship and Management Journal*, 2(3): 391-412.

Schatten, A. (1999). *Cellular Automata*. At: http://www.ifs.tuwien.ac.at/~aschatt/info/ca/ca.html#Introduction.

Schelling, T. (1978). *Micromotives and Macrobehavior*. New York: Norton.

Spelta, A. M. (1603). *Historia de fatti notabili occorsi nell'universo, & in particolare del regno de' Gothi, de' Longobardi, de i Duchi di Milano … dall'anno VL sino al MDIIIC*, Pietro Batoli Ed..

Sugden, R. (1989). Spontaneous Order. *Journal of Economic Perspectives*, 3(4), 85-97.

Toffoli, T., & Margolus, N. (1987). Cellular Automata Machines: a New Environment for Modeling. Cambridge, MA: The MIT Press.

Ulam, S. M. (1991). *Adventures of a mathematician*. Berkeley, CA: University of California Press.

Zuradelli, C. (1888). *Le torri di Pavia*. Pavia: Fusi.

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