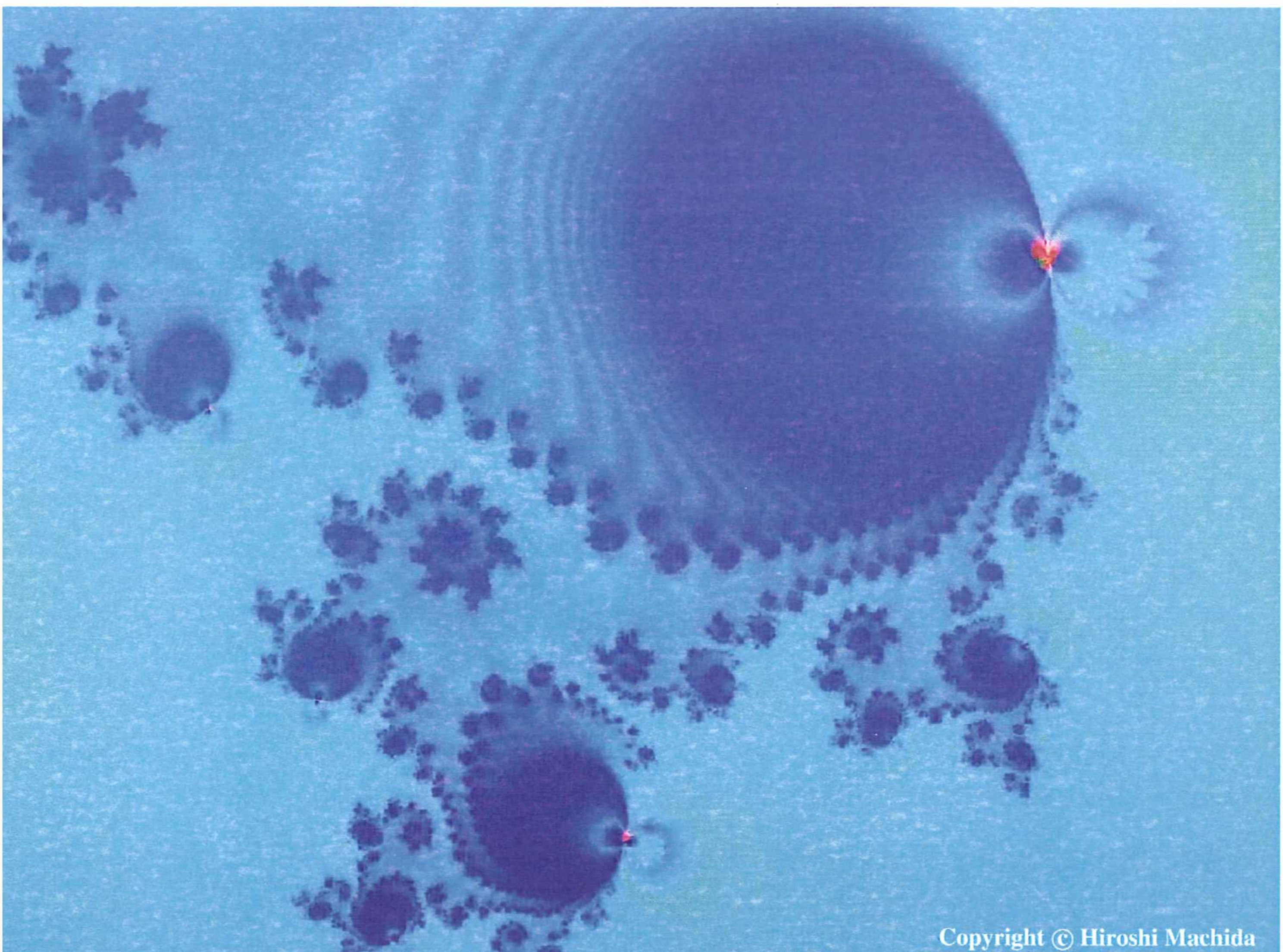
The 6th International Conference

# COMPLEX SYSTEMS 2002

## <Complexity with Agent-based Modeling>

September 9-11, 2002
Chuo University, Tokyo, Japan




**Editors**
Akira Namatame, David Green, Yuji Aruka, Hiroshi Sato

# Proceedings of

## The 6th International Conference on

# COMPLEX SYSTEMS (CS02)

**-Complexity with Agent-based Modeling-**

September 9-11, 2002
Chuo University, Tokyo, Japan

*In cooperation with*

The Third International Conference on Discrete Chaotic Dynamics in Nature and Society

**Acknowledgement**

(1) This conference in part is jointly organized by the project team of Grand-in -Aid for Scientific Research ( C ) ( 1 ) No. 14580486,Japan Society for the Promotion of Science (JSPS) .
This publication is also financially supported by this aid from JSPS.
(2) The session on Experimental Economics is supported by the Open Research Center: Experimental Economics: A new method of teaching economics and the research on its impact on society, The Graduate School of Economics, Kyoto Sangyo University *(www.kyoto-su.ac.jp/project/orc)*.

# Table of Contents

# Complex Systems vs. Simplex Systems

## *The Behaviour of Collectivities following the Combinatory System View*

**Piero Mella**
Faculty of Economics, University of Pavia, ITALY
Via San Felice, 5 - 27100 Pavia - Phone: +39.0382 506263 - Fax (office): +39.0382 506228
www.ea2000.it/mella - Email: piero.mella@unipv.it

## Abstract

In Agent-Based Models, collectivities are normally interpreted as complex (adaptive) systems, defined as a plurality of (usually large) blind (reactive) or intelligent (active) specialized (usually strongly) interacting agents (or processes), whose collective macro behaviour - determined by the interaction of the micro behaviours of the agents - is non-linear and derives from local (proximity) rules following a schema (innate or learned). My paper also aims to demonstrate that collectivities whose agents show a *similar* nature or significance, develop *analogous* micro behaviours which produce analogous effects and are not (necessarily) interconnected can produce a complex (self-organized or even chaotic and, of course, path-dependent) macro behaviour: the accumulation of objects, the spread of features or information and the pursuit or exceeding of a limit. I have provocatively defined these collectivities as *simplex systems*, since the similarity of the agents and the micro behaviours, and the absence of direct interactions among the agents, make these collectivities a particular simplified class of complex (adaptive) systems as usually conceived. When simplex systems show a *micro-macro feedback* between micro and macro behaviours, they can then be viewed as *Combinatory Systems*. The second aim of this research paper is to illustrate, in particular – with the aid of a *combinatory lattice* – the systems of improvement and progress, whose effect is to produce progress in the overall state of a simplex system in which the agents pursue their search for individual improvement, as we can typically observe in collectivities of economic agents moved by their own interests or objectives in a local and global context.

**Keywords:** agent-based systems, combinatory systems, populations and collectivities, path dependence, chaos in social dynamics.

## 1 The study of collectivities and the Sciences of Complexity. The macro approaches (a short survey)

Collectivities have always been a very complex subject of study, and for this reason a fascinating and interesting one as well.

If observed from a certain distance collectivities appear distinct with respect to the individuals they are composed of, and thus seem able to show an autonomous macro behaviour due to the joint action of the micro behaviours of the agents. This macro behaviour may show a chaotic dynamic or a regular one as a result of some kind of self-organization.

Originally, the study of collectivities considered as systems of agents followed the traditional *macro* or *analytic* approach, which produces a macro description of the behaviour of collectivities only following general macro rules and ignoring the micro behaviour of the agents.

Within the Sciences of Complexity the macro approach is typical of Population Dynamics Models, which try to represent population behaviour (increase, evolution, co-evolution and competition) in terms of the number of their elements, using, for example, Malthusian models and Volterra-Lokte equations in various forms (Volterra 1931, Ardeni and Gallegati 1999).

Wiener's Cybernetics (Wiener 1948, von Foerster 1960, Haken 1977, Kauffman 1993) and, in particular, Evolutionary Cybernetics (Campbell 1960, Gould 2000), are other macro approaches which aim to explain how collectivities are able to arrange their components to form patterns different or better than the previous ones.

Von Bertalanffy's General System Theory (von Bertalanffy 1968) and Haken's Synergetics (Haken 1977), Forrester's Systems (Industrial) Dynamics approach (Forrester 1961), Senge's System Thinking approach (Senge 1990), and Maturana's and Francisco Varela's Autopoiesis approach (Maturana and Varela 1980, Varela 1979 and 1981, Maturana and Guiloff 1980, Zeleny 1981) offer powerful conceptual frameworks and practical tools for building models of the behaviour of collectivities.

## 2 Collectivities as Complex Systems. The micro approaches (a short survey)

Since Thomas Schelling's attempt, in his very famous work, *Micromotives and Macrobehavior*, to offer through game theory and the prisoner's dilemma model a logical explanation of why collective macro behaviour derives from the micro behaviours of intelligent agents (Shelling 1960 and 1978), and Conway's discovery of the fantastic world of Life (Gardner 1970), the study and simulation of the behaviour of collectivities or of agents (Harding 1990) has followed micro or internal or synthetic approaches.

In Agent-Based Models, collectivities are normally interpreted (Flake 1998) as Complex (Adaptive) Systems (Coveney and Highfield 1995, Mitleton and Kelly 1997, Allen 1997, Axelrod 1997, Goldspink 2000), defined as a plurality (usually large) of blind (reactive) or intelligent (active) multi-character (Drogoul and Ferber 1994), specialized, usually (strongly) interconnected (Wu 1997, Granovetter 1974, Grimmett 1999) interacting agents (or processes) (Holland 1995, Gell-Mann 1994&1995, Stacey 1995), often showing possible multi-level hierarchies (Chan 1998, Gaffeo 1999, Cummings and Staw 1985: 2) whose collective macro behaviour is determined by the interaction of the micro behaviours of the agents (Otter, Veen and Vriend 2001) on the basis of simple local rules (Waldrop 1993) according to a schema (innate or learned) (Dooley 1997, di Primio 1999), and which shows non-linear dynamics (Lewin 1992)[1] as well as unanticipated global properties, or patterns (Foster and Metcalfe 2001: 4).

The Complex Adaptive Systems approach, in particular (Allen 1997), studies how collectivities interact and exchange information with their environment to maintain their internal processes over time through adaptation, self preservation, evolution and cognition (in the sense of Maturana and Varela 1980: 13), and to achieve collective decisions (Rao and Georgeff 1992: 127-146, Wooldridge and Jennings 1994) within a relational context of micro behaviours (Conte and Castelfranchi 1992).

> *"If you want to understand why a person acts as she does, it is certainly possible to look around in the immediate environment for an explanation. But often an explanation needs to look also, or perhaps primarily, at events that occurred in the past and at how the present situation developed from previous circumstances."* (Gilbert 1995).

The analysis of complex systems implies a Recursive Approach, and two of the most powerful tools are represented by the Cellular Automata Theory – introduced in the late 1940's by John von Neumann (Burks 1966), which allows the researcher to explore complex systems by simulating Artificial Life (Alife) (Liekens 2000)– and the Genetic Algorithms approach (Bak 1994, 1996, Schatten 1999).

The theory of Cellular Automata builds mathematical models of a system whose agents are represented by cells in an array (a lattice) of one or more dimensions (Creutz 1996, Schatten 1999). It is important to note that the rules that define the micro behaviour of a cell are only local rules, in the sense that the state of the cell depends only on one of a specified number of neighbours and not on the state of the array (Gardner 1970, Toffoli and Margolus 1987, Dewdney 1989 and 1990, Ulam 1986 and 1991).

Following the logic of cellular automata, many fundamental instruments have been created to simulate Artifical Societies (Resnick 1994, Epstein and Axtell 1996, http://zooland.alife.org). Among the most well-known are Dorigo's Ants approach (Dorigo, Di Caro and Gambardella 1999, Hölldobler and Wilson 1990), Langton's Swarm approach (http://www.swarm.org), Reynolds's boids (Reynolds 1987), and Dolan's Floys approach (Dolan 1998).

These instruments also demonstrate that there is also a hidden order in the behaviour of collectivities of simple living autonomous reactive agents.

As Holland attempts to demonstrate, the most powerful approach to understanding and showing the hidden order in collective behaviour is the genetic algorithms approach (Holland 1975) and the related genetic programming approach of Koza (Goldberg 1989, Koza 1992).

## 3 Towards simplex systems

Concentrating on the micro approaches, I observe that if, on the one hand, it is easy to explain (perhaps properly speaking, to describe), assuming only local rules, the behaviour of a flock of birds, a school of fish, or a herd of elephants when these collectivities have already formed, or the spread of information, the imitation of choices (information contagion), or the percolation effects in probabilistic diffusion systems (Frey and Decker 1996, Grimmet 1999), on the other hand it is not so easy to apply this micro approach to describe, for example, the grouping of flocks (a bird is attracted by the flock and not by its neighbours), swarms, herds and other collectivities, the formation of graffiti on walls (people are attracted by the cloud of graffiti and not by the behaviour of other people), the breaking out of applause (many people applaud if the applause dies down), or the phenomenon of a rising murmur in a crowded room.

It is clear that a person who is talking raises his voice to go beyond the increasing murmur of the crowed room only for individual necessity, and not because his neighbours are raising their voices, or that a fish joins a school of fish because of the presence of a predator, and only if he can perceive the school, and not because he sees other fish join the school.

Similarly, it is hard to explain, by exclusively using local rules, the exceeding of limits (all people park or drive fast even in the presence of parking limits and speed limits), the pursuit of records, the eternal maintenance of feuds, and the phenomenon of urban settlements.

The analysis and understanding of these and many other phenomena, which will be mentionned below, is even more difficult because they often are "one way" and cannot be repeated or reproduced, as if due to chance.

While the phenomenon of urban settlements appears to repeat itself many times, even with particular variations, the same cannot be said for the construction of towers in medieval Pavia, which is an amazing event because it is unique.

And while the pursuit of a record is a common event, it is more difficult to see any similarity with feuds, which are usually so particular as to defy comparison.

In many cases, moreover, Agents cannot observe the collectivity, and thus their neighbours, and must act only based on individual necessities, as in the case of the formation of piles of garbage (if I need to throw away a piece of garbage and I see a garbage pile, I prefer to leave my garbage behind), of annoying and dangerous wheel ruts on the highway (passing trucks need to maintain their trajectory on the carriageways, and this is reinforced by these micro behaviours), or of paths in fields (people prefer to cross a field where a path is visible), and so on.

In all these circumstances, the Agents' micro behaviours seem to follow some necessitating macro variable(s) deriving from the collectivity (the cloud of graffiti, the pile of garbage, the applause, the carriageway, the feud, and so on) rather than obey a set of local rules.

## 4 Peculiarities of Simplex Combinatory Systems

I think that these and many other interesting phenomena, or effects, might be attributed to the basic behaviour of a simple kind of collectivity made up of Agents (or elements) which:

- show a similar nature, structure or significance[2];
- develop analogous micro behaviours which produce analogous effects;
- are not necessarily interconnected by evident interactions, or by network, web or tree structures;
- perceive some macro variable (or a set of variables) related to the macro behaviour (or the macro effect) of the collectivity as a whole;
- can evaluate, in a simple pay-off table, positive or negative gaps (advantages or disadvantages) in their status or performance with respect to the macro variable;
- take individual micro decisions (by a process of imitation and social learning) in order to increase (if positive) or reduce (if negative) the perceived gaps,
- but these decisions recursively change the value assumed by the macro variable, and this modifies the perceived positive or negative gaps, driving the agents to adapt their behaviour by new decisions.

These collectivities constitute a particular class of complex systems (Gell-Mann 1994:18) but as they follow the simplest schema of adaptation, and because of the similarity of agents and behaviours, the absence of organizational or social links, levels, specializations, multidimensionality and, particularly, direct interactions, cooperation or competition among the agents and their neighbourhood, we could provocatively define these collectivities as a *simplex system*.

The operative logic of simplex systems is as basic as their structure:

- on the one hand, the macro behaviour of the system, as a whole, derives from the combination (defined in an opportune way) of the analogous micro behaviours and effects of its similar agents;
- on the other hand, the macro behaviour determines, or conditions, or directs the subsequent micro behaviours;
- this reciprocal relationship may be defined as micro-macro feedback and this produces the simplest level of adaption of the entire system (Gell-Mann 1994:20).

Because the micro behaviours, combined together, produce the macro behaviour (and the macro effect) that, in turn, conditions the micro behaviours of the agents, according to a feedback relation between micro and macro behaviours, these systems can also be conceived of as (a particular class of) *Combinatory Systems*.

I firstly observe that simplex or combinatory systems show various forms of self-organization, in the sense that the agents may adjust and specialize their micro behaviours and produce a macro behaviour that can lead to some macro phenomenon, macro effect, or recognizable pattern, even without any interaction among the components.

The four main classes of such phenomena are: the accumulation of objects, the spread of features or information, the pursuit of an objective or the exceeding of a limit, and the interdependent dynamics of individual improvement and collective progress in the overall state of a collectivity (defined in opportune ways).

If we accept the traditional definition of self-organization as the macro behaviour of a collectivity of agents in which the micro behaviours appear to be directed, or organized, by an Invisible Hand, or Supreme Authority, in order to produce the emerging phenomenon represented by the formation of ordered structures, of recognizable patterns (Foster and Metcalfe 2001: 130, Pelikan 2001), then all the above-mentioned collective phenomena can also be defined as self-organization or spontaneous order (Sugden 1989, Kauffman 1993, Ashford 1999, Swenson 2000).

## 5 Combinatory Automaton

In order to simulate simplex or combinatory systems and to produce the macro effects that characterize *simplex systems*, it is useful to build a *Combinatory Automaton*, based on the following definition (Fig. 1):

1. a set of **N** *cells* $\mathbf{A_i}$, $1 \leq i \leq N$ – arranged in a ***combinatory lattice*** $\Lambda$ – characterized by a variable $\mathbf{a_i(t)}$ defined in a domain $\mathbf{d_i} \in \mathbf{R}$; each cell may be considered as an Agent of the corresponding *combinatory system*;
2. the ***analytical state*** of the automaton, $\Lambda(\mathbf{t_h}) = [\mathbf{a_i(t_h)}]$ is defined as the values $\mathbf{a_i(t_h)} \in \mathbf{d_i}$ assumed by $\mathbf{A_i}$ for each $\mathbf{t_h} \in \mathbf{T}$ (we assume **T** is a discrete time scale); the *time series* $\mathbf{A_i(T)} = [\mathbf{a_i(t_0), a_i(t_1), a_i(t_2), \dots}]$ represents the *micro behaviour* of the agent $\mathbf{A_i}$ in period $\mathbf{T}$[3];
3. the ***synthetic state*** of the automaton at $\mathbf{t_h}$ is defined as the value assumed by a global *macro variable* $X(\mathbf{t_h}) = \mathbf{C}_{1 \leq i \leq N}\, \mathbf{a_i(t_h)} = \mathbf{C}[\Lambda(\mathbf{t_h})]$ derived from a *combination* of those values, where $\mathbf{C}_{1 \leq i \leq N}$ indicates a set of *combination operation(s)*, appropriately specified (sum, product, average, min, max, etc.), of values associated with $\Lambda(\mathbf{t_h})$;
4. the ***output behaviour*** of the automaton at $\mathbf{t_h}$ is defined as the value assumed by the *variable*, $X(\Lambda, \mathbf{t_h}) = \mathbf{F}\ \{X(\mathbf{t_h})\}$; the *recombining function* **F** (or *macro rule*) transforms the ***synthetic state*** into the ***output*** of the automaton; the *time series* $X(\Lambda, \mathbf{T}) = [X(\Lambda, \mathbf{t_0}), X(\Lambda, \mathbf{t_1}), X(\Lambda, \mathbf{t_2}) \ \dots]$ of $\Lambda(\mathbf{t_h})$ represents the *macro behaviour* of the corresponding *combinatory system* in the period **T**; in many simple cases, $X(\Lambda, \mathbf{t_h}) = X(\mathbf{t_h})$;
5. the ***output effect*** of the automaton at $\mathbf{t_h}$ is defined as the value assumed by the *variable*, $E(\Lambda, \mathbf{t_h}) = \mathbf{G}\{X(\Lambda, \mathbf{t_h})\}$; the *function* **G** transforms the ***output behaviour*** into the ***output effect*** of the automaton; in many cases, when the combinatory system show only the macro behaviour, we assume $\mathbf{E}(\Lambda, \mathbf{t_h}) = X(\Lambda, \mathbf{t_h})$;
6. at time $\mathbf{t_{h+1}}$ each $\mathbf{A_i}$ changes its value following the *micro* ***transition function:*** $\mathbf{a_i(t_{h+1})} = \mathbf{f_i}\ \{\mathbf{N_i}[\mathbf{a_i(t_h)}, X(\Lambda, \mathbf{t_h})]\}$ where $\mathbf{N_i}$ represents the ***decision variable*** – appropriately specified (difference or variation) – resulting from a set of *necessitating factors* which push the agent $\mathbf{A_i}$ to modify the previous values $\mathbf{a_i(t_h)}$ according to the ***output variable***, $X(\Lambda, \mathbf{t_h})$ (or the ***output effect*** $E(\Lambda, \mathbf{t_h})$); in many simple cases, $\mathbf{N_i} = [\mathbf{a_i(t_h)} +/- X(\mathbf{t_h})]$;
7. for the recursive dynamics being produced we

must assume to be also specified the ***initial state*** $\mathcal{A}(t_0)$;

8. as a result, a general *micro-macro feedback* relation connects the micro state to the macro state through the *variable* $X(\mathcal{A}, t_h)$ (or $E(\mathcal{A}, t_h)$) which may be thought as an *organizing* or ***driving variable*** of the corresponding *combinatory system* because it determines the subsequent micro behaviour of the agent $A_i$;
9. the *set of rules* specifying the operations $\mathbb{C}_{1\le i\le N}$ and $\mathbb{N}_i$ and the rules **F** and $f_i$, represent the ***operative programme***, which produces the dynamics of the *combinatory automaton*.

The definition is summarized in the formal model:

$$
[A]\begin{cases}
\mathcal{A}(t_0) = [a_i(t_0)] \leftarrow \text{"CHANCE"} & 1\le i\le N \quad [A.1] \\
X(t_h) = \mathbb{C}_{1\le i\le N}[a_i(t_h)] = \mathbb{C}[\mathcal{A}(t_h)] & h=0,\ 1,\ 2,\ .. \quad [A.2] \\
X(\mathcal{A}, t_h) = F\ \{\ X(t_h)\ \} & [A.3] \\
E(\mathcal{A}, t_h) = G\ \{\ X(\mathcal{A}, t_h)\ \} & [A.4] \\
a_i(t_{h+1}) = f_i\ (\mathbb{N}_i[a_i(t_h), X(\mathcal{A}, t_h)) & 1\le i\le N \quad [A.5] \\
\text{Set: } \{\ \mathbb{C}_{1\le i\le N}, \mathbb{N}_i, F, G, f_i \text{ and } g_i\ \} & \text{programme} \quad [A.6]
\end{cases}
$$

The combinatory automaton may be:

a) *stochastic*, if a probability, $p_i$, is associated with the transition of state of each $A_i$, $1\le i\le N$; in the opposite case it is *deterministic* (in the model I have not explicitly considered probabilities); probabilities may be: *fixed* if: $p_i \equiv p$ for every **i** and **j**; *time dependent* if $p_i \equiv p(t_h)$; *time and agent dependent* if $p_i \equiv p_i(t_h)$; *output dependent* if $p_i \equiv p_i(X, t_h)$;

b) *time-response sensitive*, if the length of the *period of transition of state* $\Delta t_i = t_{i+1} - t_i$ is agent-output dependent; $\Delta t_i \equiv \Delta t_i(X, t_h)$; (in the model I have not explicitly considered sensitivity in the time response because I have assumed $\Delta t_i$ = constant);

c) *two dimensional* if Agents are arranged in **R** rows and **C** columns, so that **N** = **(R*C)**, or *multidimensional* (in the model I have considered a mono-dimensional automaton);

d) *mono* or *multiple-driven*, depending on the number of driving variables $M_j(\mathcal{A}, t_h)$ (in the model I have considered a mono driven automaton and j=1 is omitted);

e) *reversible*, if $a_i(t_h) = a_i(t_k)$, h<k, is admitted; (in the model I have not explicitly considered reversibility).

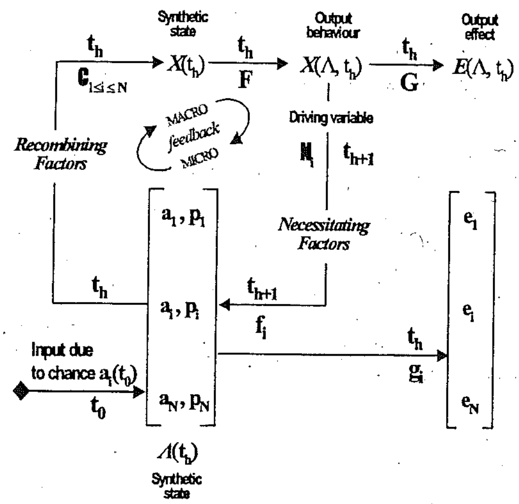



*Fig. 1 – A model of combinatory automaton*

## 6 Chaos and Path dependence in Combinatory Automata. The role of reversibility

In *stochastic combinatory automata* when both *probabilities* $p_i \equiv p_i(X, t_h)$ and *periods* of transition of state $t_i \equiv t_i(X, t_h)$ are agent/time/state sensitive, the *micro behaviours* are conditioned by the *macro behaviour* of the entire system, which makes the *micro-macro feedback* evident.

Probabilities can act in two ways:

a. as *stop-or-go probabilities*, in the sense that if the probabilistic event occurs, the agent assumes a new state; otherwise, it maintains its actual state. We might symbolise this type of probability on writing: $p_i(X, t_h)_{[0,1]}$. "0" means that if the event does not occur, the agent maintains its state; "1" that the agent changes its state if the event occurs;
b. as *transition probabilities*, in the sense that if the probabilistic event occurs, then the agent enters a new state; if the event does not occur, the agent assumes a different state or returns to the past one. We might write: $p_i(X, t_h)_{[-1,1]}$.

The *social combinatory systems* that are most interesting and easiest to represent are the *irreversible* ones (build a tower or not, teach Italian or English to babies). In these systems both the micro and macro behaviours produce permanent effects that may be viewed as increasing or decreasing cumulative processes in which probabilities are: $p_i(X, t_h)_{[0,1]}$.

*Chaos* arises in *combinatory systems* when the *hypothesis of reversibility* is introduced (for example:

to speak or to keep quiet in the next minute, wear a skirt or miniskirt on different days, choose road A or B on different days) (Fuchs and Haken 1989). These systems are generally governed by *transition doors* probabilities: $\mathbf{p}_i(X, \mathbf{t}_h)_{[-1,1]}$.

When *reversibility* in micro behaviours or in micro effects is possible, the combinatory system's macro behaviour, or macro effect, can show a cyclical dynamic and, under certain conditions concerning the probability function regarding the transition of state of the elements, a chaotic one as well, where no cycles are recognizable in the orbit of the system starting from random initial values (Gleick 1988).

Examples of *reversible* systems are those of diffusion and dissemination (fashion and contagion), whose elements may at different times present the same state chosen from a *repertoire* (Lustick 2000).

In particular we can note that in *probabilistic reversible combinatory systems* both the *random initial states* of the system and the *probability function of transition of states*, which depend on the macro behaviour at each iteration, can be determined with ample approximation.

These hypotheses of randomness in the initial conditions and in their evolution as well (*history dependence*), together with the imprecision of the measurement of the micro behaviours, produce dynamic instability in the macro behaviour and explain almost all the cases of path dependence, both in reversible and, in many cases, irreversible systems, as we can argue from [A.1] in the previous models (Liebowitz and Margolis 1998, Arthur 1988 and 1994).

## 7 A simple Combinatory Automaton. The Murmur and Noise in crowded rooms.

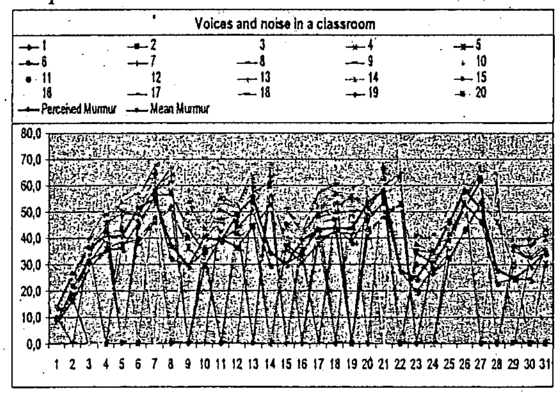
Let us first consider the phenomenon of a murmur arising in a crowded room, typically produced by a simplex system. The murmur is the output of the crowded room considered as a combinatory automaton and is produced by the combination of the voice levels of the individual speakers who, in order to make themselves heard, must raise their voices some decibels above the murmur. But recursively this increases the murmur, in a typical feedback between micro and macro behaviour.

We can represent this phenomenon through the combinatory automaton **[B]**[4]:

$$
\textbf{[B]}\quad
\begin{cases}
v(n, t_0) \leftarrow \text{“CHANCE”} & 1 \le n \le N \\
\mathbf{M}(t_h) = \{\, \mathbf{k}\,(1/N)\, \Sigma_{1\le n\le N}\, [v(n, t_h)] + Q\, r_p(t) \,\}\,(1 - \mathbf{a}) & h = 0, 1, 2, \ldots \\
v(n, t_{h+1}) = [w(n)\, \mathbf{M}(t_h) + (v_{min}(n) + v_{max}(n)\, l_p(n, t_h))\,]\, s_p(n)\, b_R(n, t_h) & 1 \le n \le N
\end{cases}
$$

The simulation model of Figure 2 shows this phenomenon; it describes a linear automaton of 20 (non-ordered) speakers observed for 30 iterations.

**Test 1** – *External noise Q = 5 dec. Mean probability to speak = 87%*




**Test 2** – *External noise Q = 10 dec. Mean probability to speak = 90%*

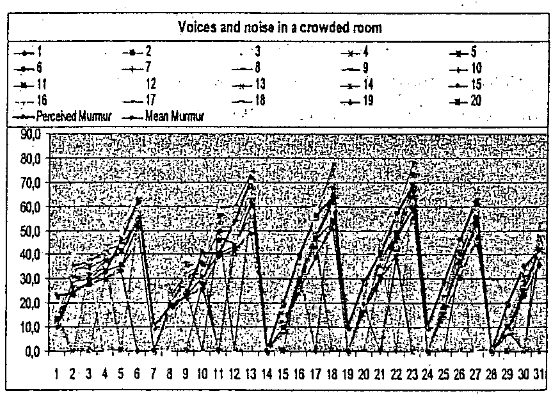



***Fig. 2*** - *Model of Murmur and Noise system with 20 agents and differentiated probabilities for each agent.*

The voice levels (coloured lines) are the variables associated with the speaking agents.

The noise (bold blue line) may be viewed as the output of the combinatory automaton constituting the collectivity considered as a whole.

The crowded room recombines the voice levels into a simple mean $\Sigma_{1\le n\le N}$ [v(n, $t_h$)]), but the level of noise also depends on several factors: the nature of the speakers, the necessity of speaking, the structure of the room that, recombining the voices, can maintain or reduce the murmur, which specify a set of appropriate parameters for the macro and micro functions F and f.

In particular, the *necessity to speak* is represented by a probability that may or may not depend on time and on the number of talking people. If we introduce tolerance in the model, that is the maximum level of bearing, then the system may show a cyclical behaviour.

In conclusion, the talking agents thus seem self-organized to simultaneously raise their voice level and produce a stable, or a rising or fluctuating noise, a typical pattern which, I am sure, we have experienced several times.

## 8 The main ideas of Combinatory System Theory

On the basis of the previous example, we can introduce the Combinatory Systems Theory which offers guidelines for observing collectivities as simplex or combinatory systems[5].

The central idea is that we can view a collectivity as a combinatory system only if the agents are similar and their analogous micro behaviours are not exclusively determined by local rules but also by a general micro-macro feedback rule (Foster and Metcalfe 2001: 132-133) which acts over many cycles[6]. The combinatory systems approach is neither a macro approach nor a micro approach; it is a micro-macro approach[7].

The macro behaviour – or the associated macro effects – may be thought of as a dynamic *director* or, better yet, as an internal dynamic *organizer* which seems to direct, or organize, the individual behaviours to adapt their micro behaviours to the macro behaviour in order to produce the collective phenomena (von Foerster 1960, Haken 1977, Prigogine 1985, Kauffman 1993, Martelli 1999).

If the micro behaviours of the agents are determined exclusively by the macro behaviour, the combinatory system is a pure simplex system.

If they depend also on an opportune neighborhood, as well as, naturally, on the macro behaviour, the combinatory system is characterized by incomplete and limited information.

Finally, if the agents' behaviour depends only on local rules acting on a defined neighborhood, without considering any micro-macro feedback, the system is a complex system and loses the characteristics of a combinatory system[8].

The second main idea is that in order to understand and explain the activity of any combinatory system we must recognize the nature of both the recombining factor and the necessitating ones, whose joint action gives rise to and maintains the macro and micro behaviours.

The third main idea is that the starting up of a combinatory system – even if its behaviour is deterministic - also requires a random input to activate the micro-macro feedback. The output is then entirely determined by the structural dynamics of the system, according to the micro and macro rules and the micro-macro feedback[9].

Combinatory systems are *recursively closed systems*; their dynamic is only due to the joint action of "chance" and "necessity"; they might thus also be called "chance-necessity" systems [10].

Other relevant characteristics (I will only mention these) concern the fact that, even though combinatory systems are unorganized and closed systems, they can organize themselves into specialized subsystems and show ramifications (Monod 1971, Maturana and Varela 1987), and can expand their effects on elements belonging to a vaster environment.

## 9 A Combinatory Automaton simulating Improvement and Progress

A very special and important combinatory system is the one I have named the *Improvement and Progress Combinatory System*, since its particular effect is to produce progress, according to commonly accepted value judgements regarding an improvement in the overall state of a collectivity.

When "by chance" an improvement begins in one or all of the agents of the system, then "by necessity" progress occurs throughout the system; the improvement spreads and the progress continues, until a limiting state is reached in which no further improvement can be carried out and no further progress can occur.

To simulate the systems of IMPROVEMENT AND PROGRESS we can build a Combinatory Automaton following these very simple rules according to the general definition in par. 6. In particular:

1) the ***analytical state*** of the automaton, $\mathcal{A}(t_h)=[\mu_i(t_h)]$ is defined as the values of the parameter of improvement $\mu_i(t_h)$ associated with

each $\mathbf{A_i}$ in $\mathbf{t_h \in T}$; the *time series* $[\boldsymbol{\mu}_i(\mathbf{t}_0), \ldots, \boldsymbol{\mu}_i(\mathbf{t}_h), \ldots]$ represents the *improvement path* of $\mathbf{A_i}$;

2) the ***synthetic state*** and the ***output*** of the automaton at $\mathbf{t_h}$ are defined as the value assumed by the *macro variable* $\pi(\mathit{A}, \mathbf{t_h}) = (1/\mathbf{N})\ \Sigma_{1\leq i\leq N}\ \boldsymbol{\mu}_i(\mathbf{t}_h)$, which represents the parameter of progress for the entire system; the *time series* $[\boldsymbol{\pi}(\mathit{A}, \mathbf{t}_0), \ldots, \pi(\mathit{A}, \mathbf{t}_h), \ldots]$ represents the *progress path* of $\mathit{A}(\mathbf{t}_h)$;
3) at time $\mathbf{t_h}$ the *necessitating operation(s)*, which condition the internal event (decision) that determines the agents' behaviour, derives from the difference $\mathbf{N}_i = \Delta\boldsymbol{\mu}_i(\mathbf{t}_h) = \boldsymbol{\mu}_i(\mathbf{t}_h) - \pi(\mathit{A}, \mathbf{t}_h)$, which denotes the *deviation* between the *individual improvement* level and mean level denoting *collective progress*; so that each $\mathbf{A_i}$ perceives an *inferiority*, with respect to the mean, if $\Delta\mu(n, t) < 0$, or a *superiority* in the opposite case and acts to maintain or increase the advantage, or eliminate or reduce the disadvantage;
4) the *micro transition* functions can assume the following expression; $\boldsymbol{\mu}_i(\mathbf{t}_{h+1}) = \{\ \boldsymbol{\mu}_i(\mathbf{t}_h) + \mathbf{p}_i\ \mathbf{i}_i\ \Delta\boldsymbol{\mu}_i(\mathbf{t}_h)\ \}$; the probabilities $\mathbf{p}_i = \mathbf{p}(\Delta)_{[0,1]}$ (the probability is not only agent-dependent but is assumed to be dependent, for each agent, on the sign of $\Delta\boldsymbol{\mu}_i(\mathbf{t}_h)$) represent the *necessitating factors* under the hypothesis of irreversibility (agents can only ameliorate their improvement measures), or $\mathbf{p}_i = \mathbf{p}(\Delta)_{[-1,1]}$ under the hypothesis of reversible micro behaviour (agents may also reduce their improvement measures with respect to the progress measure); the parameter $\mathbf{i}_i$ indicates the random action of $\mathbf{A_i}$ to ameliorate his performance;
5) to make the model more general, I have also supposed that the *micro transition* functions which determine the change in the agents' behaviour depend also on *environmental factors,* that is on *external events*, whose general expression is: $\{\mathbf{r}_i\ [\mathbf{k}\ \boldsymbol{\mu}_i(\mathbf{t}_h) + \mathbf{h}\ \pi(\mathit{A}, \mathbf{t}_h)]\}$, where $\mathbf{r}(\Delta)_{[0,1]} = \mathbf{r}_i$ (or $\mathbf{r}(\Delta)_{[-1,1]} = \mathbf{r}_i$ in the case of reversibility) indicates the probability of this external event – assumed to be dependent, for each agent, on the sign of $\Delta\boldsymbol{\mu}_i(\mathbf{t}_h)$ – and $[\mathbf{k}\ \boldsymbol{\mu}_i(\mathbf{t}_h) + \mathbf{h}\ \pi(\mathit{A}, \mathbf{t}_h)]$ represents the amount of influence of the environmental variables on the improvement measure of $\mathbf{A_i}$. This expression translates the common idea that the attempt to improve performance is conditioned both by the previous level of performance measure of the system (following the parameter $\mathbf{k}$) and by the previous level of individual performance measure ($\mathbf{k}$ and $\mathbf{h}$ are scalar coefficients, but we may normally assume that $\mathbf{h}$=0).
6) the micro and macro dynamics are thus connected, since the level of improvement measures determines the level of the progress, but this in turn modifies the subsequent improvement variables in the typical *micro-macro feedback;*

The combinatory automaton is summarized in the formal model:

$$[\mathbf{C}]\left\{\begin{array}{l|ll} \boldsymbol{\mu}_i(\mathbf{t}_0) \leftarrow \text{“CHANCE”} & 1\leq i\leq N & [C\text{-}1] \\ \pi(\mathit{A}, \mathbf{t}_h) = (1/N)\ \Sigma_{1\leq i\leq N}\ \boldsymbol{\mu}_i(\mathbf{t}_h) & h=0, 1, 2, \ldots & [C\text{-}2] \\ \boldsymbol{\mu}_i(\mathbf{t}_{h+1}) = \{\ \boldsymbol{\mu}_i(\mathbf{t}_h) + \mathbf{p}_i\ \mathbf{i}_i\ \Delta\boldsymbol{\mu}_i(\mathbf{t}_h)\ \} & 1\leq i\leq N & [C\text{-}3] \\ \quad + \{\ \mathbf{r}_i\ [\mathbf{k}\ \boldsymbol{\mu}_i(\mathbf{t}_h) + \mathbf{h}\ \pi(\mathit{A}, \mathbf{t}_h)]\ \} & & \\ \Delta\boldsymbol{\mu}_i(\mathbf{t}_h) = \boldsymbol{\mu}_i(\mathbf{t}_h) - \pi(\mathit{A}, \mathbf{t}_h) = \mathbf{N}_i & 1\leq i\leq N & [C\text{-}4] \end{array}\right.$$

and, due to the structure of [C-2], we can name it the MEDIAL AUTOMATON of IMPROVEMENT AND PROGRESS.

Two important remarks need to be made.

At first I observe that, even if the MEDIAL AUTOMATON of IMPROVEMENT AND PROGRESS is quite general, we may conceive of two other different automata simply by specifying equation [C-2].

1. Assuming:

$$\pi(\mathit{A}, \mathbf{t}_h) = \mathbf{Max}_i\ \boldsymbol{\mu}_i(\mathbf{t}_h) = \boldsymbol{\mu}^M(\mathbf{t}_h)$$

we have defined the *Maximal* Automaton of Improvement and Progress (or "of *pursuit*"); consequently, in equation [C-4],

$$\Delta\boldsymbol{\mu}_i(\mathbf{t}_h) = \boldsymbol{\mu}_i(\mathbf{t}_h) - \boldsymbol{\mu}^M(\mathbf{t}_h)$$

represents the *quantum of inferiority* perceived by each agent compared with the improvement parameter of the *leader agent* (or "the best"). We thus witness micro behaviours aimed at reducing the inferiority with respect to the level of progress, and this causes a macro behaviour whose effect is to raise the average level of improvement, so that some agents manage to further raise the previous level of progress.

2. Assuming:

$$\pi(\mathit{A}, \mathbf{t}_h) = \mathbf{Min}_i\ \boldsymbol{\mu}_i(\mathbf{t}_h) = \boldsymbol{\mu}_m(\mathbf{t}_h)$$

we have defined the *Minimal* Automaton of Improvement and Progress (or "of *flight*"); in equation [C-4]:

$$\Delta\boldsymbol{\mu}_i(\mathbf{t}_h) = \boldsymbol{\mu}_i(\mathbf{t}_h) - \boldsymbol{\mu}_m(\mathbf{t}_h)$$

represents the *quantum of superiority* perceived by each agent compared with the improvement parameter of the *base agent.*

These systems act in a symmetrical way with respect to the previous ones, since each agent of the system tries to outdistance as much as possible its own level of improvement from the level of progress, to flee from the minimum level of improvement, to increment its own superiority. This leads to a general increase in the average level of improvement, which ends up raising the parameter of progress, further boosting the levels of improvement.

Secondly, let us note that if both $\mathbf{p}_i$ and $\mathbf{r}_i$ admit reversibility, then the system is *strong reversible*; if only one of the two probabilities admits reversibility (generally $\mathbf{r}_i$), the system is *weak reversible*; elsewere, it is *irreversible* and improvement and progress are continuously increasing.

Let us assume a *Maximal* Automaton of Improvement and Progress of ten agents described by Figure 3, which also shows the dynamics of this system under different hypotheses of reversibility.

As we can easily note, the more reversibility is introduced, the more the macro and micro behaviours are chaotic, as we can verify by simulating dynamics for 20 iterations.

Among the phenomena that can be explained using the system of improvement and progress are the growth of productivity in firms, the continuous improvement in the quality of products, progress in the sciences and in technology, and the evolution of all types of species as a consequence of individual choices (Mella 2001).

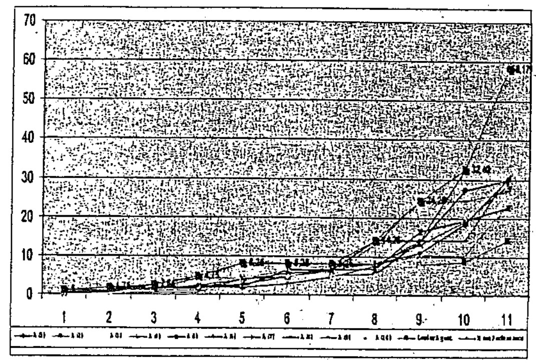
## 10 Conclusions and challenges

The Combinatory System Theory focuses attention on the importance of both the micro-macro feedback and of the necessitating and recombining factors that produce and maintain it.

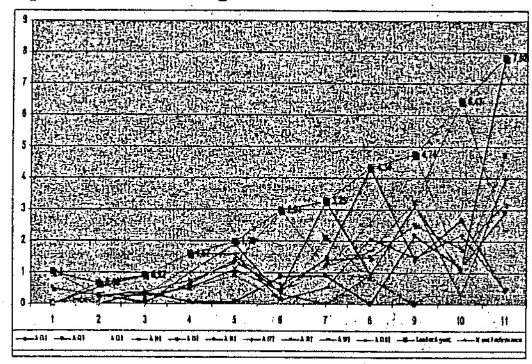
With the aid of unsophisticated Combinatory Automata I have also tried to demonstrate that even simplex systems, in which the reversibility in micro behaviours and effects is admitted, may show cyclical, irregular and even chaotic behaviour.

The challenge of Combinatory System Theory is threefold: (i) to develop more general and further sophisticated Combinatory Automata for any specific class of combinatory system; (ii) apply the theory to understand collectivities operating in the real world; (iii) specify, for any real observed collective phenomenon, the sets of necessitating and recombining factors which allow us to interpret and control the collectivity that produces it.

**Test 1 – Irreversible** *Maximal* Automaton of Improvement and Progress




**Test 2 – Weak Reversible** *Maximal* Automaton of Improvement and Progress




**Test 3 – Strong Reversible** *Maximal* Automaton of Improvement and Progress

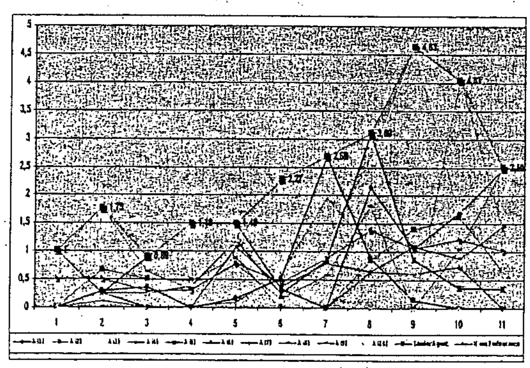



***Fig. 3** - Maximal Automaton of Improvement and Progress (10 agents, 10 iterations) with different hypothesis on probabilities*

## 11 References and web sites

**Allen** P.M. (1997), Cities & Regions As Self-Organizing Systems: Model of Complexity, Environmental Problems & Social Dynamics Series, Vol 1, Gordon & Breach Science Pub
**Ardeni** P.G. and **Gallegati** M. (1999), Fluctuations and growth due to technological innovation and diffusion, in Gallegati and Kirman
**Arthur** W.B. (1988), Self-Reinforcing Mechanisms in Economics, in Anderson and Arrow and Pines

**Ashford** N (1999), Spontaneous Order, in Ideas on Liberty, Vol. 49, No. 7, at: http://www.fee.org/freeman/99/9907/ashford.html
**Axelrod** R. (1997), The Complexity of Cooperation, Princeton University Press
**Bak** P., (1994), Self-Organized Criticality: A Holistic View of Nature, in Cowan, Pines and Meltzer
**Bak** P. (1996), How Nature Works: The Science of Self-Organized Criticality, Springer, Berlin
**Burks** A.W. (1966), (Ed.), Theory of Self-Reproducing Automata [by] John von Neumann, University of Illinois Press, Urbana
**Campbell** D.T. (1960), Blind Variation and Selective Retention in Creative Thought as in Other Knowledge Processes, Psych. Rev. 67/380
**Chan** D. (1998), Functional Relations Among Constructs in the Same Content Domain at Different Levels of Analysis: A Typology of Composition Models, Journal of Applied Psychology, 83, 234-246
**Conte** R. and **Castelfranchi** C. (1992), Mind is not enough: precognitive bases of social interactions, Proceedings of Symposium on Simulating Societies
**Coveney** P. and **Highfield** R. (1995), Frontiers of Complexity, Faber and Faber
**Creutz** M. (1996), Cellular Automata and self organized criticality, in: http://ttt.lanl.gov/PS_cache/hep-lat/pdf/9611/9611017.pdf
**Cummings** L.L. and **Staw** BM (eds.) (1985), Research in Organizational Behaviour,7, Greenwich, CT: JAI Press (1-37)
**Deneubourg** J.L. and **Goss** S (1989), Collective Patterns and Decision Making, Ethology, Ecology and Evolution I
**Dewdney** A.K. (1990), The Magic Machine, WH Freeman, New York
**Dewdney** A.K. (1989) The Turing Omnibus, Computer Science Press, New York
**Di Primio** F. (1999), Role of symmetry in robot (group) behavior, Workshop, GMD, AiS, 28.10.1999
**Dolan** A. (1998), Floys. New members in the Artificial Life zoo, at: http://www.aridolan.com/eFloys.html
**Dorigo** M., **Di Caro** G., **Gambardella** L.M. (1999), Ants Algorithms for discrete optimization, in Artificial Life, , at: ftp://iridia.ulb.ac.be/pub/mdorigo/journals/IJ.23-alife99.pdf
**Drogoul** A. and Ferber J. (1994), Multi-agent simulation as a tool for studying emergent processes in societies, in Gilbert/Doran
**Epstein** M. J. and Axtell R. (1996), Growing Artificial Societies, Social Science from the bottom up, MIT Press
**Forrester** J.W. (1961), Industrial Dynamics, MIT Press, Cambridge
**Foster** J. and **Metcalfe** J.S. (2001), Frontiers of Evolutionary Economics. Competition, Self-Organization and Innovation Policy, J. Foster and J.S. Metcalfe Eds., Edward Elgar Publishing, Cheltenham Glos, UK
**Foster** J. and **Metcalfe** J.S., (2001), Modern Evolutionary economic perspectives: an overview, in Foster and Metcalfe
**Frey** T. and **Decker** E. (1996), Percolation Theory, Ecological Complexity Seminar: Fall, at: http://algodones.unm.edu/~chdecker/complexity/96fall/percol.html#1
**Fuchs** A. and **Haken** H. (1989): The synergetic approach to pattern recognition: Irreversible Processes and Self-Organization, W .Ebeling, H Ulbricht, eds., Teubner Verlag, Leipzig
**Gaffeo** E. [1999], Tutorial on social interaction economics, in Gallegati and Kirman
**Gardner** M. (1970), Mathematical Games, Scientific American 223(4), October
**Gell-Mann** M. (1995/96), Complexity, Vol. 1, no.5 ©, in: http://www.santafe.edu/sfi/People/mgm/complexity.html
**Gilbert** N. (1995), Simulation: an emergent perspective, in New Technologies in the Social Sciences, at: http://www.soc.surrey.ac.uk/research/simsoc/tutorial.html
**Gleick** J. (1988), Chaos making a new science, Cardinal, London
**Goldberg** D.E. (1989), Genetic Algorithms in Search, Optimization and Machine Learning, Addison Wesley Publishing Co
**Goldspink** C. (2000), Modelling social systems as complex: Towards a social simulation meta-model Journal of Artificial Societies and Social Simulation vol. 3, no. 2, 31 March 2000, in: http://www.soc.surrey.ac.uk/JASSS/3/2/1.html
**Gould** S. (2000), The Theory of Options: A New Theory of the Evolution of Human Behavior, uPUBLISH.com
**Granovetter** M.S. (1974), Getting a job: a study on contacts and careers, Cambridge, Harvard University Press
**Grassé** P.P. (1959), La reconstrucion du nid et les coordinations inteindividuelles chez BellicositermesNatalensis et Cubitermes. La théorie de la stigmergie : essai d'interpretation du comportement des termites constructeurs, in Insect Sociaux, 6, pp. 41-48
**Grimmett** G. (1999), Percolation, Second Edition, Springer Sciences on Line
**Haken** H. (1983), Advanced Synergetics, Springer-Verlag
**Haken** H. (1977) Synergetics: An Introduction, Springer-Verlag
**Holland** J.H. (1975), Adaption in natural and adaptive systems,University of Michigan Press, Ann Arbor
**Holland** J.H. (1995), Hidden Order: How Adaptation Builds Complexity, Perseus Books, Cambridge, Massachusetts
**Hölldobler** B. and **Wilson** EO (1990) The Ants. Belknap/Harvard University Press
**House** R, **Rousseau** D.M. and **Thomas-Hunt** M. (1995), The Meso Paradigm: A Framework for the Integration of Micro and Macro Organizational Behavious, in Research in Organizational Behaviour, Cummings and Staw (1995), eds., 17, Greenwich, CT, JAI Press, 71-114
**Kauffman** S. (1993), The Origins of Order: Self-Organization and Selection in Evolution. Oxford University Press
**Koza** J.R. (1992), Genetic Programming. On the programming computers by means of natural selection, The MIT Pess
**Liebowitz** S.J. and **Margolis** E. (1998), The New Palgrave's Dictionary of Economics and the Law, MacMillan,
**Liekens** A. (2000), Artificial Life?, in http://alife.org/index.php?page=alife&context=alife
**Lustick** S. (2000), agent-based modelling of collective identity: testing constructivist theory, Journal of Artificial Societies and Social Simulation , , vol. 3, no. 1, in: http://www.soc.surrey.ac.uk/JASSS/3/1/1.html
**Martelli** M. (1999), Introduction to discrete dynamical systems and chaos, J. Wiley & Sons, N. Y.
**Maturana** H.R. and **Varela** F. (1980), Autopoiesis and Cognition, Reidel
**Maturana** H.R. and **Varela** F. (1987), The Tree of Knowledge. New Science Library
**Maturana** H.R. and **Guiloff** G. (1980), The quest for the intelligence of intelligence, J. Social Biol. Struct. 3: 135-148
**Mella** P. (2001) Self-organization in collectivities. The Combinatory System approach: www.ea2000.it/cst/genindex.htm
**Mitleton-Kelly** E. (1997), Complex Adaptive Systems in an Organizational Context. "Organizations as Co-evolving Complex Adaptive Systems", British Academy of Management Conference, in: http://bprc.warwick.ac.uk/eve.html
**Monod** J. (1971), Chance and Necessity, An Essay on the Natural Philosophy of Modern Biology. Knopf, New York
**Nicolis** G. and **Prigogine** I. (1989), Exploring Complexity: An Introduction, Freeman & Co.: New York
**Otter** H.S., **van der Veen** A and **de Vriend** HJ (2001), ABLOoM: Location behaviour, spatial patterns, and agent-based modelling, J. of Artificial Societies and Social Simulation vol. 4, no. 4
**Pelikan** P. (2001), Self-organizing and Darwinian selection in economic and biological evolutions: an enquiry into the sources of organizing information, in Foster and Metcalfe (2001)
**Prigogine** I. (1985), New perspectives on complexity. In: The Science and Praxis of Complexity. United Nations Library
**Prigogine** I. and **Stengers** I (1984), Order Out of Chaos (La Nouvelle Alliance - Les Metamorphoses de la Science) Bantam: New York
**Rao** A. and **Georgeff** M. (1992), Social plans: preliminary report. In Decentralized AI 3 – Proceedings of MAAMAW'91, E Werner and C Castelfranchi (Ed.), Elsevier North Holland
**Reinolds** C. W. (1987), Flocks, herds and schools: a distributed behavioral model. Comp. Graph., 21(4), 25-34
**Resnick** M. (1994), Turtles, termites and traffic jams. Explorations in massively parallel microworlds, Cambridge, MA: MIT Press
**Rousseau** D.M. (1985), Issues of Level in Oganizational Research, in Research in Organizational Behaviour, LL Cummings and BM Staw eds. (1985), 7, Greenwich, CT: JAI Press, 1-37
**Schatten** A. (1999), Cellular Automata, in: http://www.ifs.tuwien.ac.at/~aschatt/info/ca/ca.html#Introduction
**Senge** PM (1990), The Fifth Discipline: The Art & Practice of The Learning Organization, Currency Doubleday, New York,

**Shelling** T. (1960), The strategy of conflict, Cambridge, Harvard University Press

**Shelling** T. (1978), Micromotives and Macrobahavior, Norton, NY, Toronto

**Stacey** R.D. (1995), The science of complexity: an alternative perspective for strategic change processes, Stratecig Management Journal, Vol. 16

**Sugden** R. (1989), Spontaneous Order, Journal of Economic Perspectives 3:4, Fall, pp. 85-97

**Swenson** R. (2000), Spontaneous Order, Autocatakinetic Closure, and the Development of Space-Time, Annals New York Academy Of Sciences, vol. 901 (pp. 311-319), at: http://evolution.philosophyofscience.net

**Toffoli** T. and Margolus N (1987), Cellular Automata Machines: a New Environment for Modeling. The MIT Press

**Ulam** S.M. (1991), Adventures of a mathematician, Berkeley

**Ulam** S.M. (1986), Science, computers, and people, Basel

**Varela** F. (1981), "Describing the logic of the Living. The adequacy and limitations of the idea of Autopoiesis" in Zeleny (1981)

**Varela** F. (1979), Principles of Biological Autonomy, North Holland, Amsterdam

**Volterra** V. (1931), Leçons sur la Théorie Mathématique de la Lutte pour la Vie, Gauthier-Villars, Paris

**Von Bertalanffy** L. (1968), General System Theory. New York, George Braziller

**Von Foerster** H. (1960), On the Self-Organizing Systems and Their Environments. Cameron M. CY, Self-Organizing Systems. Oxford

**Waldrop** M.M. (1993), Complexity: The Emerging Science at the Edge of Order and Chaos, Simon and Schuster, New York

**Wiener** N. (1948), Cybernetics. or Control and Communication in the Animal and Machine, MIT Press, Cambridge On the, (2nd ed.1961)

**Wolfram** A. (1984), Cellular Automata as Moldels of Complexity, Nature, 311/4, 419-424

**Wolfram** S. (1994), Cellular Automata and Complexity, Reading, Addison-Wesley, Mass., 1994

**Wooldridge** M. and Jennings NR (1994), Toward a theory of cooperative problem solving, in Proceedings of the 6th European workshop on modelling autonomous agents in a multi-agent world, Odense, Denmark, august

**Wu** J. (1997), Introduction to percolation theory, at: http://garnet.berkeley.edu/~jqwu/paper1/paper1.html

**Zeleny** M. (1981), Autopoiesis, a theory of living organization, Elsevier, North Holland, New York

**Zollo** G., **Iandoli** L. and **De Maio** L., (2001), An application of an Ant Colony System to the Analysis of Organizational Learning Processes, in VIII. SIGEF Congress Proceedings, New Logics for the new economy, Naples, 2001

---

[1] So that the system's history is irreversible and the system's future unpredictable, or even chaotic, if the description of regularities is impossible (in the sense of Gell-Mann 1995, of Wolfram's classification scheme, 1984, 1994, or of Devaney 1989).

[2] *Combinatory systems* differ from *complex adaptive systems* (CAS) in many aspects. *Firstly*, because *combinatory systems* do not necessarily present phenomena of adaptation but, generally, some form of self-organization due to the *micro-macro feedback*, that is the adaptation of agents to a synthetic variable produced by the macro behaviour of the system. Adaptation may be a characteristic of some particular class of CS representing populations and not only collectivities. A *second* difference is observable also as regards the similarity of the agents: *"Here we confront directly the issues, and the questions, that distinguish* CAS *from other kinds of systems. One of the most obvious of these distinctions is the diversity of the agents that form* CAS. *Is this diversity the product of similar mechanisms in different* CAS? *Another distinction is more subtle, though equally pervasive and important. The interactions of agents in* cas *is governed by anticipations engendered by learning and long-term adaptation."*. (Holland 1995: 93). The *third* main difference regards the absence of interactions among the agents; in combinatory systems agents generally interact only with some macro variable and not each other. The *fourth* relevant difference is that the theory of CAS observes the macro effects of the system produced by the agents that follow a schema or change the schema previously followed. Any micro-macro feedback between the micro behaviours and the schema is considered as a relevant characteristic. *Finally*, ignoring the micro-macro feedback implies that CAS theory only focuses its attention on *necessitating* factors and ignores the *recombining* ones (see par. 8).

[3] In many cases a set of function $g_i$ transform each $a_i(t_h)$ in a different variable $e_i(t_h) = g_i\,[a_i(t_h)]$ that may represent the micro effect of the micro behaviour of agent $A_i$, $1 \le i \le N$; in many simple cases, when the $A_i$ shows only the micro behaviour, we may assume $e_i(t_h) = a_i(t_h)$.

[4] All the models may be algebraically rearranged and simplified.

[5] *Combinatory System Theory* is presented at: www.ea2000.it/cst.

[6] Of course, the fundamental *micro-macro feedback* may also be accompanied by several other loops which make the system's dynamics non-linear. I have preferred to mention only the micro-macro fundamental feedback that may be specified in appropriate micro-macro loops.

[7] We can consider the *micro-macro feedback* approach as a *meso* approach (Rousseau 1985, House, Rousseau and Hunt 1995). *Combinatory System Theory* follows the bottom up approach of Epstein and Axtell (1996), but unlike that approach ours considers the micro-macro feedback as the origin of self-organization in Combinatory Systems.

[8] For this reason we cannot in general consider the ants, the swarm and, more generally, the cellular automata approaches as examples of combinatory systems, except in the case where the macro behaviour may affect the micro behaviours of the agents in some way, for example by creating an "aromatic potential field" by spreading *pheromones* or other permanent messages. With their micro behaviours the agents spread pheromone across one site (micro information); the increasing concentration of pheromone (global or macro information) increases the probability that each agent will move in the direction of that site. The micro-macro feedback is quite evident (Zollo, Iandoli and De Maio 2001, Deneubourg and Goss 1989). This behaviour is the consequence of *stigmercy* (Grassé 1959).

[9] We must thus remember that in order to produce the micro behaviours (and observe the macro behaviour) we must usually supply *energy* to the system. Since the main objective of the theory of combinatory systems is to bring out the operative logic typical of such systems, in order to simplify their description energy inputs are usually not considered, in part because such considerations are usually superfluous, if not impossible. In order to give a technical explanation of the action of such systems, in particular for purposes of designing them, knowledge of the energy inputs can be indispensable.

[10] We have used, though with a different meaning, the same terminology used by Monod (1971), who, in his famous *Chance and Necessity* examined a very powerful combinatory system: that leading to a dynamic evolution in a population due to random mutations produced in the DNA that "by necessity" spread as a result of the invariant reproductive mechanism of cells.

Haken also speaks of *chance* and *necessity* when he proposes constructing models of complex systems. Here Haken considers *chance* as the unpredictable fluctuation from an unstable equilibrium state, and *necessity* as the movement towards a new, more stable state (Haken 1983, Prigogine and Stengers 1984).

Chance will not only set under way the macro behaviour but will also determine the direction, that is the direction of the "winning" fluctuation. Prigogine bases his theory on the emergence of order in complex systems on the consequences of fluctuations (Nicolis and Prigogine 1989, Haken 1983).

A simple way to observe the inflow of the random fluctuations in orientating the *direction* of the "macro" dynamics of combinatory systems - even if it is not sufficient to describe the effect of *chance* on the overall dynamics of a combinatory system - is offered by the *Polya Urns* and by the *Ehrenfest Urns*.